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Research

Effect of Teaching Mathematics Supported by Problem-posing Strategies on Problem-posing Skills

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Abstract:

This study investigates the effects of teaching mathematics supported by problem-posing strategies on fourth-grade students' problem-posing skills. This study also seeks to determine students' views about the process. To this end, the study employed an explanatory sequential design, a mixed method that incorporates collecting quantitative and qualitative data. The study group consisted of fourth-grade students studying in two different classrooms of a public school in the west of Türkiye in the 2021–2022 academic year. Data were collected through a "Problem-Posing Skills Test" and a "Semi-Structured Interview Form." The research concludes that teaching mathematics supported by problem-posing strategies improves students' problem-posing skills. In addition, this method was more effective than the one used in the control group in developing students' structured, semi-structured, and free problem-posing skills. At the end of the interviews, it was determined that teaching mathematics supported by problem-posing strategies was an innovative, student-centered, and emotionally stimulating technique. It was also found that the students had more difficulty in the semi-structured and free problem-posing tasks.

Keywords: Problem-posing strategies, problem-posing skills, student views, primary school, mixed methods

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INTRODUCTION

Achieving mathematical competence depends on the systematic and logical presentation of several complex processes (Turkish Ministry of National Education [MoNE], 2018). Therefore, designing effective mathematical activities and teaching effective mathematics are as important as content. Effectively conveying mathematical activities requires designing learning environments equipped with various features (Cobb vd., 1992; Erdem & Soylu, 2019).

Today's curriculum emphasizes the need to conduct lessons with methods that present every subject effectively and permanently (MoNE, 2018; National Council of Teachers of Mathematics [NCTM], 2000). Studies show that permanent learning is more easily achieved in learning environments where students play an active role and have fun (Güneş et al., 2011; Karasu Avcı & Ketenoğlu Kayabaşı, 2019). In this context, important studies have been conducted recently using different methods (Erdem & Soylu, 2019; Güneş vd., 2011; Şengül & Dereli, 2013), such as digital content applications (Nuha et al., 2018; Özgen et al., 2019; Papadakis et al., 2021), game-supported studies (Başün & Doğan, 2020; Bilgin, 2021; Çilingir Altıner, 2018), and interdisciplinary approaches (Akben, 2018; 2019; Macun, 2019). These studies show that lessons conducted using different methods that address students' cognitive and affective domains are essential for developing students' mathematical skills.

Problem-posing is a unique mathematical activity that helps improve cognitive and affective competencies (Cai & Leikin, 2020). Problem-posing is defined as reformulating a given problem or producing new problems or questions (English, 1997; Silver, 1994) and is considered an important intellectual activity that constitutes an integral part of school mathematics and a balanced mathematics curriculum (Hansen & Hana, 2015).

In traditional teaching settings, students solve problems given in textbooks using methods introduced by teachers. Students are rarely asked to pose problems and solve them (Korkmaz & Gür, 2006). However, the problem-posing skill, which is at the center of curricula implemented in many countries such as the USA, Australia, China, and Türkiye, is seen as an important mathematical skill that students should develop (Australian Education Council [AEC], 1991; Ministry of Education of the People's Republic of China [MoE], 2011; MoNE, 2018; NCTM, 2000). In addition, researchers stated that students' success in problem-solving was related to their problem-posing abilities, and they used problem-posing as a measure of such learning outcomes (Cai & Hwang, 2020; Cai et al., 2013; Silver & Cai, 1996).

Problem-posing becomes a learning activity when students pose problems according to their own interests, whereas it becomes a teaching method when teachers pose a problem for students to solve (Stoyanova, 2003). In this context, Silver (1994) stated that problem-posing can be applied in three different ways in fulfilling the problem-posing tasks given to students: a) before solving a problem (different and unique problems from the existing problem are posed), b) during problem-solving (a solved problem is reformulated), and c) after solving the problem (objectives and terms of the current problem are changed). Stoyanova and Ellerton (1996) group these tasks into three groups: structured, semi-structured, and free problem-posing tasks. Christou et al. (2005), based on the findings of Stoyanova and Ellerton (1996), created a problem-posing model that includes four processes: editing, selecting, understanding, and translating. According to the classification of Stoyanova and Ellerton (1996), in the structured problem-posing strategy, students are given a well-structured problem or a problem solution, and they are asked to create a new problem related to the given problem or its solution. In the semi-structured problem-posing strategy, students are given an open-ended situation; then, they are asked to construct new problems using their knowledge, skills, previous mathematical experience, and concepts. In the free problem-posing strategy, students are given a real-life situation and asked to pose problems without any limitations.

It is seen that three aspects of problem-posing, namely ‘construct,’ ‘variable,’ and ‘intervention,’ have been addressed in studies on problem-posing. Problem-posing as a structure is related to what a problem-posing activity includes, its type, and what features it should contain to be considered an activity (Cai & Hwang, 2020; Cansız Aktaş, 2022). For example, in a problem-posing activity, the data collected in the problem sentence, its subject, context, expression, whether it is solvable or not, and whether the problem can be reformulated or not, in short, the elements related to the nature of the problem constitute the structure of problem-posing (Koichu, 2010). The class teacher’s reformulation of a problem to include a more challenging feature based on an issue they have solved is related to the structure of problem-posing. For this, the teacher should think of a structure that includes more complex processing steps. In studies considered as a structure, problem-posing is observed and defined using various methods, interviews, and discourse analysis (Kılıç, 2014). How teachers understand, learn, and pose problem-posing is investigated by examining the problems they pose (Koichu & Kontorovich, 2013). It is tried to understand what the students think while posing a problem. The relationship between teachers’ problem-posings and students’ problem-posings is investigated.

Problem-posing as a variable can be defined as a well-defined and measurable feature that allows for comparison with other variables (e.g., creativity, problem-solving). In problem-posing as a variable, features such as the number, originality, and difficulty levels of the problems posed by students can be used as criterion to determine the level of other skills (Ayvaz & Durmuş, 2021; Cansız Aktaş, 2022; Cai & Hwang, 2020; Mallart et al., 2018). In literature, there are more studies in which problem-posing is considered a variable. In these studies, measurements related to problem-posing are made, and the relationship of these measurements to other skills is explained. For example, the effect of problem-posing on variables such as problem-solving, creativity, and mathematics achievement is examined. In these studies, it was found that students’ problem-solving and problem-posing skills improved in lessons that continued with problem-posing tasks (Akay, 2006;

Silver & Cai, 1996; Turhan & Güven, 2014; Xie & Masingila, 2017). Problem-posing is the best tool to observe the three indicators of mathematical creativity (fluency, flexibility, and originality) (Roble et al., 2021; Silver, 1994). Students who deal with different problems have reduced dependance on textbooks (Çomarlı & Gökkurt Özdemir, 2019) and have a positive impact on mathematical literacy and self-efficacy beliefs (Geçici & Aydın, 2019; Liu et al., 2020; Özgen, 2019). It has developed a positive attitude toward mathematics as it strengthens conceptual understanding (Akay & Boz, 2010; Katrancı & Şengül, 2019).

Finally, in studies that address the 'intervention' aspect, problem-posing tasks are included in the learning process (Cai & Hwang, 2020; Liljedah & Cai, 2021). Including problem-posing as a teaching method in the lesson plan aims to develop and expand the understanding of problem-posing (Cai & Hwang, 2020; Cansız Aktaş, 2022; Li et al., 2020; Zhang & Cai, 2021). This includes problem-posing in course practice as a teaching method, such as question and answer, discussion, lecture, and drama. Changing the current teaching method and integrating problem-posing to develop students' creativity will create new learning opportunities, and its results will be effective (Leikin & Elgrably, 2020). In studies where problem-posing is addressed as an intervention, problem-posing improves digital skills, such as computer programing, by increasing cooperation. In addition, it has been observed to enable students to have higher self-efficacy and lower cognitive load (Wang & Hwang, 2017). It has been determined that teacher training can be conducted through problem-posing (Cai & Hwang, 2020; Li et al., 2020; Passarella, 2021), and integrating these activities into primary school mathematics teaching can improve mathematical creativity (Bicer et al., 2020). However, difficulties encountered by teachers through problem-posing (organization, designing, evaluation, quality problem-posing, negative impact on exams) and students (low-quality problem-posing, lack of experience, language use, lack of confidence) were also revealed (Li et al., 2020; Xie & Masingila, 2017). When these studies are examined, the task of integrating problem-posing as an intervention into mathematics lessons is seen by researchers as a new phenomenon, and there are studies in the world that try to include problem-posing in various educational levels (Bicer et al., 2020; Brown & Walter, 1983; Li et al., 2020; MoE, 2011; Zhang & Cai, 2021).

As can be understood from the studies dealing with different aspects of problem-posing, problem-posing is an important activity that contributes to students' mathematical skills as both a skill and a method. Based on these findings, mathematics teaching should incorporate the problem-posing method. However, there are a limited number of studies in the literature in which problem-posing is used as an instructional model. Örnek and Soylu (2021) developed a six-step problem-forming learning model. This model was designed to create a common approach in learning environments to teach problem-posing and develop problem-solving skills. The model was tested on pre-service elementary mathematics teachers using a pretest-posttest comparison group design. At the end of the application, it was determined that this model improved conceptual learning, positively affected the solvability of problems, and ensured the correct use of mathematical language and grammar rules. Zhang and Cai (2021) examined 22 teaching practices of teachers who used problem-posing as a teaching method to support students' mathematics learning. The findings obtained at the end of the study are remarkable. First, problem-posing did not sufficiently penetrate the intended curriculum level (textbooks and materials). Second, it is related to how to organize the problems constructed by students. Students create problems related to the topic and unexpected problems that arise. Third, teachers who learned to teach using problem-posing could develop problem-posing teaching practices even after two years. According to the results obtained, more detailed research is needed to identify relevant and irrelevant problems, the difficulty levels of relevant problems, and how teachers handle these problems. Cai et al. (2020) investigated the impact of a problem-posing workshop on mathematics teachers' understanding of problem-posing and lesson design. While none of the teachers used any problem-posing components in their lesson plans before the seminar, they included them in more than 80% of their lesson plans after the workshop. Jia and Yao (2021) analyzed six versions of Chinese textbooks to identify how problem-posing has emerged in these materials over the years. According to the results of this study, problem-posing activities have only been systematically and purposefully incorporated into textbooks recently. However, even now, it is stated that books contain very few problem-posing activities. In another study, problem-solving, and problem-posing skills of sixth-grade children on "decimal fractionswere improved with the problem-posing approach. According to the results, this approach improved students' problem-solving success and problem-solving skills (Turhan, 2011). Fifth-grade students engaged in structured and semi-structured problem-posing activities prepared for the acquisition of "solving and constructing problems requiring operations with natural numbers" for five weeks. It was determined that problem-posing activities increased students' success in problem-posing and solving (Şakar, 2018). Teachers who learned to teach mathematics through problem-posing improved their situation-posing performance and beliefs about teaching through problem-posing after participating in three workshops (Li et al., 2020). Using 101 pre-service teachers, the effects of the problem-posing approach used in science education on students' problem-solvings and metacognitive awareness were investigated. The study results showed that structured, semi-structured, and free problem-posing activities improved students' problem-solving skills and metacognitive awareness (Akben, 2018). A quasi-experimental study conducted with 83 pre-service primary school teachers determined that problem-posing-based practices in teaching the concept of mole effectively increased both the problem-solving skills and academic achievement of pre-service teachers. Because of these studies, it was seen that the problem-posing approach can also be used in science teaching (Akben, 2019). Therefore, although there are extensive studies on problem-posing strategies in various educational settings, there are a limited number of studies focusing on the effects of mathematics teaching supported by problem-posing strategy, especially in the primary school context, and revealing students' perspectives on the process. In this study, a quasi-experimental research was designed, and mathematics instruction supported by problem-posing strategies was conducted. The effect of this instruction on the problem-posing skills of fourth-grade elementary school students was examined, and a second qualitative phase was designed for this method. In this qualitative phase, we presented the findings by considering students' opinions.

Problem of the Study

In this context, this study examines the effect of teaching mathematics supported by problem-posing strategies on the problem-posing skills of primary school fourth-grade students and analyzes the students' views on this method. The research questions were as follows:

1. Does teaching mathematics supported by problem-posing strategies significantly affect students' problem-posing skills?

2. What are the students' views on teaching mathematics supported by problem-posing strategies?

METHOD

Research Model

This study employed one of the mixed research methods, the "Explanatory Sequential Design," which includes collecting quantitative data and then collecting qualitative data to elaborate on the quantitative data. In this design, quantitative data are more important than qualitative data, and the aim is to support quantitative results with qualitative findings. (Creswell & Plano-Clark, 2018).

In line with this design, in the first stage of the research, one of the quasi-experimental designs, the "nonequivalent control group pretest-posttest design," was used (Büyüköztürk et al., 2014). In this design, which is used in many studies, especially in the field of education, which of the groups will be the experimental group and which will be the control group is decided by random assignment. The design includes an experimental and a control group. At first, a pretest was applied to the experimental and control groups. In this study, the experimental group was taught mathematics supported by problem-posing strategies, whereas the control group received no additional application. After the application, a posttest was applied to both groups, and the study ended (Lodico et al., 2006).

In the second stage, the "Case Study Design," one of the qualitative research designs, was used. A case study seeks to explore one or more phenomena, settings, programs, social groups, or interconnected systems in depth (McMillan, 2000). In this context, after completing the teaching of mathematics supported by problem-posing strategies, interviews were conducted with the students to obtain their views on the effectiveness of the process.

Participants

The school where the application would be conducted was determined using the convenience sampling method, one of the purposive sampling methods. Because many experimental studies have to use already formed groups or volunteers, it is only possible to use the convenience sampling method (Creswell, 2013). Convenience or accessible sampling is based on items that are entirely available, quick, and easy to reach. Convenience sampling is the method in which the researcher turns to the most accessible items that the researcher can obtain to form the sample from the target population, and most studies in the literature prefer this method (Baltacı, 2018). For convenience sampling, the researcher determines sufficient items from the existing items as a sample (Singleton & Straits, 2005).

The school where the study was conducted is a public school in Afyonkarahisar province in western Türkiye, which is attended by students with average academic performance and from similar socio-economic backgrounds. When deciding on the school where the study would be conducted, the researcher considered several factors, such as the school's physical features, the number of teachers and students, and whether the school administration and teachers volunteered for the study. The study included fourth graders studying in two classrooms in this school in the second semester of the 2021–2022 academic year. These two classrooms, equivalent in terms of possibilities and characteristics, were randomized into two groups (experimental and control). The research was conducted with 34 students, 17 in both groups. In both groups, 10 (58.8%) students were girls and 7 (41.2%) were boys. Therefore, the numbers of students in the experimental and control groups were very close.

The qualitative study group consisted of 8 students determined by the maximum variation sampling method, a purposive sampling technique. When using maximum variation sampling, the researcher selects some units or cases to maximize the diversity relevant to the research question. Maximum variation sampling does not seek to make generalizations; instead, it aims to capture and describe any shared theme and thus reveal different dimensions of the research problem (Yıldırım & Şimşek, 2006). To ensure diversity, the researcher identified variables such as students' mathematics achievement and gender as sources of diversity. As a result, four female and four male students in the experimental group with different mathematics achievement levels were included in the interview process.

Data Collection Tools

Problem-Posing Skills Test: The test developed by the researcher consisted of 15 problem-posing tasks. The tasks were evenly distributed as structured (5 tasks), semi-structured (5 tasks), and free (5 tasks) problem-posing tasks (Table 1). The purpose of including all three problem-posing tasks was to measure students' problem-posing skills in all three sub-dimensions. Stoyanova and Ellerton (1996) categorized problem-posing into three groups and stated that problem-posing skills can be better observed. Including these three types in the test is essential for students to recognize all variations of problem-posing types and to observe all skills called problem-posing.

Table 1

Examples of Questions in the Test

Strategies	Examples
Structured	Pose a new problem by changing the numbers, expressions, and information in the given problem or adding new information to the given problem. For a soccer match, 2524 tickets were sold on the first day and 3489 tickets on the second day. How many tickets were sold in these two days?
Semi-Structured	Using the information in the picture below, pose a problem involving subtraction. 
Free	Pose a problem with the length measurement units (mm, cm, m) in it.

The problem-posing skills test was prepared to observe the development of knowledge, concepts, and understanding of students who received formal education. To realize this purpose, an item pool was created by preparing a specification table for the gains that best measure the students' skills in all three sub-dimensions. The item pool included 38 questions (10 structured, 18 semi-structured, 10 free) covering the sub-learning areas of "Addition of Natural Numbers," "Subtraction of Natural Numbers," "Multiplication of Natural Numbers," "Division of Natural Numbers," "Length Measurement" and "Data Collection and Evaluation", considering the 4th grade learning outcomes specified by the Ministry of National Education in the mathematics curriculum.

In the development of problem situations in the question pool, mathematics textbooks belonging to public and private publishing houses, various supplementary sources (workbooks, test books, etc.), and previous studies in the literature were used (Akay, 2006; MoNE, 2018; Özgen et al., 2017; Silver & Cai, 1996). These 38 questions were sent to three faculty members who are experts in the field of mathematics teaching. Experts chose 15 questions according to the determined teaching areas (4 in addition for natural numbers, 3 in subtraction for natural numbers, 2 in multiplication for natural numbers, 2 in division for natural numbers, 3 in measuring length, 1 in data collection and evaluation), considering the cognitive development levels of 4th-grade students. In addition, these selected questions were distributed evenly according to the sub-dimensions of problem-posing. The distribution of the questions is as follows: In the structured problem-posing sub-dimension, addition, subtraction, multiplication, division, and length measurement. In the semi-structured problem-posing sub-dimension, addition (2), subtraction, length measurement, and data collection and evaluation. In the free problem-posing sub-dimension, there are questions about the sub-learning areas of addition, subtraction, multiplication, division, and length measurement. Later, these 15 problem-posing situations were submitted to eight classroom teachers who had previously taught and are still teaching fourth graders, who were asked to assess these questions in terms of their suitability for purpose. Because of the teachers' feedback, some revisions were made to the items.

Then, to test the reliability of the test, it was applied to 155 students, 75 girls (48%) and 80 boys (52%), who had characteristics similar to those of the students in the study group. Students were given two class hours to complete the test. The students' answers were scored by the researcher and two classroom teachers with at least 12 years of teaching experience based on the rubric developed by Özgen et al. (2017). Thus, interrater reliability was ensured. Then, item analysis of the test was conducted. The item difficulty index and discrimination index ranged between 0.38 and 0.75 and 0.32 and 0.59, respectively. The closer the difficulty index is to 0, the more difficult the item is, whereas the closer it is to 1, the easier it is. An item difficulty index of 0.50 indicates that the problem is moderately difficult. Items with an item discrimination index between 0.30 and 0.39 are considered quite good, while those with 0.40 and above are considered excellent (Atılgan et al., 2006; Yurdabakan, 2008). Therefore, the items in the test are moderately difficult items with good discrimination power. On the other hand, the items with item discrimination power between 0.30 and 0.39 were edited for students to better comprehend. The face validity of the test was ensured by revealing the name of the test, including sufficient descriptions for the items, and leaving enough space under the items. The Cronbach's alpha value of the test was found to be 0.89. In the problem-posing skills test, the highest and lowest scores that students can get from each item are 3 and 0, respectively. The highest and lowest scores obtained from the entire test were 45 and 0, respectively.

Semi-structured Interview Form: The form consisted of two open-ended questions to determine the participating students' views on teaching mathematics supported by problem-posing strategies. When designing the form, expert opinions were obtained from a faculty member with expertize in mathematics teaching and two classroom teachers enrolled in postgraduate education. The form was finalized by considering the experts' feedback on the questions' quality and intelligibility. In preparing the form, studies on student and teacher views on different methods used in mathematics lessons were examined. Because of the review, five questions that could be included in the interview form emerged. A faculty member working in mathematics teaching and two classroom teachers continuing their graduate education reviewed these five questions. The experts stated that two questions were out of their scope. Therefore, these two questions have been removed from the form. It was decided to use the remaining three questions in the student interviews. However, because it was observed that one question was answered by detailing the other question during the interview, this question was also removed from the test, and the interviews continued over two questions. The questions included in the form were as follows: 1. “What do you think about teaching mathematics supported by problem-posing strategies? In what way do you think it differs from traditional mathematics teaching?” 2. “In what stages/situations did you have difficulty teaching mathematics supported by problem-posing strategies?”

Data Collection

In the quantitative stage of the study, the Problem-Posing Skills test was applied as a pretest and posttest to the experimental and control groups. Students were given two class hours, and no additional time was given to any student. Before the students answered the test, they were informed that it was not intended to grade or evaluate the students; the results would be used in the study. In addition to that, students were told that they were given two class hours to answer the questions in the problem-posing test so that they would not feel time pressure. Because there were three different types of problem-posing questions in the test, students were given two class hours after answering the first questions to avoid getting bored or giving empty answers when they came to the following questions. To prevent researcher bias and not disrupt the class order that students were already accustomed to, the teachers administered the tests in both classrooms. Since the applications were decided to be carried out by the classroom teachers, a seminar on teaching mathematics supported by problem-posing strategies was given to the teacher of the experimental group for two weeks before the applications started. Then, two pilot implementations were conducted on different days to detect and correct any possible failures that may arise in the implementation of the research plan. A pilot implementation allows the researcher to control the independent variables, observe overlooked developments, recognize the changes that may occur in the application process, observe the steps of the experimental activity to be implemented, and find alternative solutions for possible problems (Teddlie & Tashakkori, 2015). In the next stage, the application process started in the experimental group (Table 2).

Table 2

Information Related to the Experimental Group Application Process

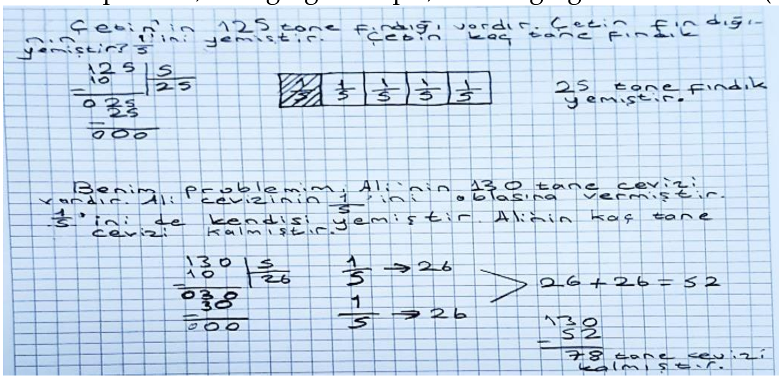
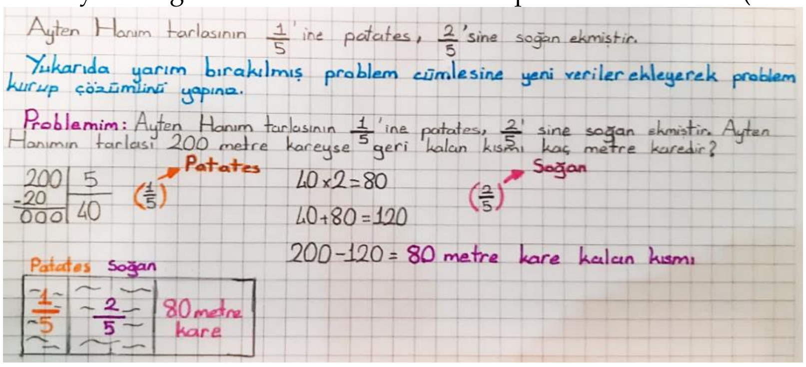
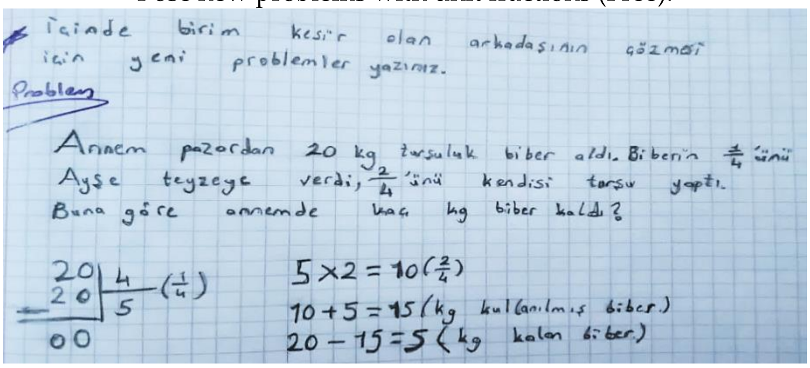
Week	Total Lessons	Strategies	Learning Outcome
Week 1	5 lessons	Structured Problem-Posing Strategy	- Defines proper, improper, and mixed fractions and expresses them in mathematical notations.
Week 2	5 lessons		- Compares and order unit fractions.
Week 3	5 lessons	Semi-Structured Problem-Posing Strategy	- Determines the specified proper fraction of a quantity.
Week 4	5 lessons		- Compares up to three fractions with equal denominators.
Week 5	5 lessons	Free Problem-Posing Strategy	- Adds and subtracts fractions with equal denominators.
Week 6	5 lessons		- Solve problems requiring addition and subtraction with fractions.

As can be inferred from Table 2, the six-week teaching of mathematics supported by the problem-posing strategies process lasted 30 class hours, one lesson per day, and five lessons per week. The application was planned for six weeks and 30 class hours because the Ministry of National Education's mathematics curriculum defines this amount of time for the objectives of the fractions subject. Therefore, because only fractions were taught, the teaching lasted six weeks. Each week, a specific learning outcome was taught to the students. Three different problem-posing strategies were used to teach these learning outcomes. The teaching of the learning outcomes with these strategies was completed in three stages: introduction, development, and evaluation. During these stages, the teachers used structured, semi-structured, and free problem-posing tasks depending on the content and presentation of the subject.

For example, in the introduction stage, after teaching the subject, the teachers asked the students to pose new problems by changing the data, information, conditions, context, etc., in the problems used in the presentation of the subject and thus completed the structured problem-posing teaching. In the development stage, semi-structured problem-posing teaching was completed by performing the problem-posing tasks in the textbook or establishing new problems using an unfinished problem situation. In the evaluation stage, free problem-posing teaching was completed by performing the problem-posing tasks developed by the teacher in advance or by performing free problem-posing tasks without any restrictions on the subject learned. A general review and evaluation of the teaching was conducted in the evaluation phase. The mistakes made by the students were identified, and necessary improvements were made. The problem-posing tasks performed by the experimental group students during the application process are shown in Table 3.

Table 3

Problem-posing Tasks Produced by Students in the Experimental Group

Examples
Çetin had 125 hazelnuts. Çetin ate $\frac{1}{5}$ of his hazelnuts. How many hazelnuts did Çetin eat? Pose new problems by changing the information given or requested in the above problem, adding new information to the question, changing the topic, or changing the conditions (Structured).  Ali has 130 walnuts. Ali gave $\frac{1}{5}$ of his walnuts to his sister. He ate $\frac{1}{5}$ of them himself. How many walnuts does Ali have now?
Ms. Ayten planted potatoes in $\frac{1}{5}$ of her field and onions in $\frac{2}{5}$ of her field. Pose new problems by adding new data to the unfinished problem statement (Semi-structured).  Ms. Ayten planted potatoes in $\frac{1}{5}$ of her field and onions in $\frac{2}{5}$ of her field. Since Ms. Ayten's field is 200 square meters, how many square meters are the rest of Ms. Ayten's field?
Pose new problems with unit fractions (Free).  My mother bought 20-kg pepper from the market. She gave $\frac{1}{4}$ of the peppers to Aunt Ayşe. She pickled $\frac{2}{4}$ of the peppers. How many kilograms of pepper does my mother have now?

As can be inferred from Table 3, in structured problem-posing tasks, after solving a problem presented by the teacher, S1 (student 1) posed a new problem similar to this problem. The student added new data to this problem and solved the new problem. In semi-structured problem-posing tasks, S8 (student 8) added new information to a problem statement left unfinished by the teacher, posed a new problem, and solved this problem. S4 (student 4) posed a new problem with unit fractions and new information in free problem-posing tasks. While problem-posing activities were carried out in accordance with all three strategies, short explanations were provided to make students understand in which cases these activities were structured, in which cases they were semi-structured, and in which cases they were free problem-posing activities. For example, when problem-posing was practiced on a solved problem, this activity was considered a structured problem-posing activity. When there is a photograph, picture, graphic, or unfinished expression in the textbook, we are told that it is a semi-structured problem-posing activity when we complete these incomplete activities. Finally, when the students reached sufficient knowledge about the subject and considered their previous problem-posing experiences, they were told that they could construct the problems they wanted. The teacher shared these explanations when necessary.

Meanwhile, traditional mathematics teaching based on textbooks continued in the control group. Apart from the problem-posing tasks in the textbook, no other task was given to the students. Some examples of problem-posing tasks in the textbook are shown in Table 4.

Table 4

Examples of Problem-posing Tasks in the Textbook

Examples	Translation
Yandaki tabloda yazılı verilerden yararlanarak bir problem kuralım. **Tablo:** Evde Beslenmek İstenen Hayvanlar **Hayvan Adı**: Kedi, Kuş, Köpek, At **Öğrenci Sayısı**: 68, 83, 124, 76 **Problem** Evde köpek beslemek isteyenlerin sayısı, kuş beslemek isteyenlerin sayısından ne kadar fazladır? Bu problemi defterinize çözünüz. Siz de tablodan yararlanarak defterinize bir problem kurunuz ve çözünüz.	Let us pose a problem using the data in the table on the side. How many more people want to have a dog at home than the number of people who want to have a bird? Solve this problem in your notebook. Using the table, you can pose a problem in your notebook and solve it.
6. Yandaki verilerden yararlanarak içinde çarpma işlemi de olan bir problem kurunuz. Kurduğunuz problemi defterinize çözünüz. 	Using the data on the side poses a problem that includes multiplication. Solve your problem in your notebook.
6. Yandaki resimden yararlanarak bir problem kurunuz. Kurduğunuz problemi defterinize çözünüz. 	Pose a problem using the picture on the right. Solve your problem in your notebook.

As seen in Table 4, the control group students also performed activities with different problem-posing strategies. In these activities, we posed new problems by reconsidering a problem or using the data in pictures, tables, and graphics. In addition, the problems were written using a amount of data and information.

Following the quantitative research process, interviews were conducted with students using the interview form. Appropriate physical conditions and sufficient time were provided for the students to express their views comfortably. One student per day was interviewed during noon breaks. The teachers of the interviewed students informed their parents in advance. Each interview took about 20-30 minutes. Before the interviews were conducted, the students were informed why this interview was being conducted and that notes would be taken during the interview. When the students were ready for the interview, they were asked to answer the questions in the interview form. While providing their answers, the students benefited from the examples in practice. The researcher noted the statements made by the students during the interview and the students confirmed them by stopping at some points during the interview. In the parts where the students did not understand or had difficulty answering, the researcher helped them explain their feelings and thoughts by providing reminder information.

Data Analysis

Quantitative data were analyzed using SPSS 26 statistical software. Parametric or non-parametric tests were applied to determine whether a statistically significant difference existed between the data obtained from different groups and two consecutive measurements of the same group. To decide on the tests to be used in the study, the normality results of the data were first examined, and the results are presented in Table 5.

Table 5

Normality test results of the tests

Test	Group	n	Shapiro-Wilk			Skewness	Kurtosis
			Statistic	df	Sig.		
Pretest	Experimental	17	0.936	17	0.269	-0.674	-0.290
	Control	17	0.893		0.053	-0.988	0.263
Posttest	Experimental	17	0.969	17	0.797	0.058	-0.610
	Control	17	0.928		0.202	-0.597	-0.390

When Table 5 is examined, it is seen that the number of students in the experimental and control groups is below 30. In cases where the number of participants is less than 30, the normality of the data is determined using the Shapiro-Wilk test (Can, 2019). The pretest values (0.269; 0.053) and posttest values (0.797; 0.202) of the experimental and control groups showed that the data were normally distributed ($p>0.05$). The distribution of skewness and kurtosis coefficients of the pretest and posttest scores of the experimental and control groups ranged -0.988 to 0.263. The skewness and kurtosis coefficients between −1 and +1 indicate the data's normal distribution (Morgan et al., 2004). Based on this information, it was decided to use t-tests, one of the parametric tests, to analyze the data because the score distributions showed a normal distribution. Whether there was a significant difference between the pre-test and post-test scores of the experimental and control groups was analyzed by independent samples t-test for the comparison of unrelated measurements and dependent samples t-test for the comparison of related measurements. In the data analysis, the significance level (p) was set at 0.05.

Qualitative data obtained from interviews with students were analyzed by content analysis. The main purpose of content analysis is to capture concepts and relations that can explain the collected data. To do this, similar data are collected under certain concepts and themes, and these are organized and interpreted in a way that the reader can understand (Patton, 2002; Yıldırım & Şimşek, 2006). The reason for conducting content analysis in the research is that it is desired to obtain some codes from the raw data obtained after the interviews with the students and to create categories from the codes. Content analysis is conducted in such cases because there are no pre-established categories. Thus, qualitative data were analyzed in four stages: coding, generating themes, reviewing themes, and defining and naming themes. In the first stage, the students' responses to each question in the interview form were analyzed to determine the conceptual meaning of their expressions, and codes were obtained. In the second stage, themes were generated on the basis of the determined codes to provide insight into the data. In the third stage, the data obtained in the first two stages were presented to the reader without including the researcher's views and comments. In the final stage, the data were interpreted, and some conclusions were made. To support the obtained data, direct excerpts from the interviews were included. The students' names in the excerpts are coded as S1, S2, S3, ..., S8. In the first stage of this four-stage cycle, determining codes from student expressions was time-consuming. Although there were sometimes dilemmas, this problem was overcome by consulting the opinions of teachers who continued their postgraduate education. The second stage can be considered the most challenging stage of this cycle because it is time-consuming for the author to reach the categories that best explain the codes. Finding and choosing the best concept to explain the codes at this stage requires intensive thinking skills. After focusing on certain concepts in this section, the best concept was reached during the article writing and editing stages. Placing the data into the categories obtained in the third stage was relatively more straightforward than that in the second. At this stage, the data were prepared in an organized manner. In the fourth stage, the meaning of the categories obtained was interpreted and presented systematically.

Validity and reliability

Internal validity refers to the degree to which observed changes or differences in the dependent variable are attributable to the independent variable. In contrast, external validity refers to the extent to which the study results can be generalized (Büyüköztürk et al., 2014). For the study results to be interpreted meaningfully, internal and external validity must be provided. For internal validity, several factors should be considered, such as history, maturation, experimental mortality, instrumentation, testing, selection bias, regression to the mean, social interaction, and attrition (Christensen et al., 2015; Creswell, 2013). Randomly selecting participants in experimental studies helps eliminate threats (Creswell, 2012).

Due to the physical conditions of the schools in Türkiye, it is often not possible to randomly assign the participants to the experimental and control groups. Therefore, in this study, the selection of experimental and control groups was performed randomly. Nevertheless, the classrooms comprise students from similar socioeconomic backgrounds. In addition, the two classrooms' first-semester mathematics grade point averages and mean scores from the pretest problem-posing skill test are very close. By including two classrooms close to each other in terms of student characteristics, selection bias and regression to the mean were controlled. No participant dropped out of school during the research, so there was no experimental mortality. The history factor was controlled because both groups did not receive any additional training during the application process. Biological and psychological changes within subjects during the research process pose a maturation threat. The careful selection of participants with similar developmental characteristics (for example, students at the same grade level) for the experimental and control groups may eliminate this problem (Creswell, 2012). The inclusion of students at the same grade level and with similar demographic characteristics in the experimental and control groups and similar developments and changes seen in both groups throughout the research indicate no maturation threat. Because of the interaction between the participants or teachers in the experimental and control groups, students in the control group may learn about the experimental process. This situation may affect the participants' scores in both groups (Creswell, 2013). To eliminate this threat, the teacher of the control group was not informed about the content of the experimental treatment. In addition, it was ensured that the experimental group's teacher did not share any information or material with the control group's teacher. The control group may feel less valuable as the experimental procedure was not applied to their groups. In this case, measures can be taken to reduce the expectations of the presumed benefits of the experimental treatment (Creswell, 2012). The control group was not informed about teaching mathematics supported by problem-posing strategies. Since the lessons in both classrooms were taught by their teachers in the order students were already accustomed to, the threats of 'diffusion of treatments' and 'compensatory rivalry' were controlled. The last threat to internal validity is testing and instrumentation. Using different instruments, such as pretests and posttests, affects participants' scores, thus threatening the experiment's internal validity (Creswell, 2013). To eliminate this threat, the same instruments were used throughout the experiment. In addition, the researcher applied instruments under similar conditions. Because the researcher did not conduct the teaching process in the groups, the researcher bias (i.e., the researcher influencing the results) was controlled. Including three types of problem-posing in the problem-posing test to increase internal validity can be added to the factors that increase internal validity. Lessons were conducted using three different problem-posing activities, and assessments were made to measure these three skills. However, the fact that the students selected for the groups were not selected impartially in the selection of the participants and that the studies were conducted only on fractions can be considered factors that reduce internal validity. In controlling this effect, the fact that the groups' previous mathematics achievements and the test's pretest scores are similar increases the internal validity. In addition, in future studies, conducting studies at different grade levels using various subjects can be considered as measures to strengthen internal validity.

Threats to external validity include the interaction of selection and treatment, setting and treatment, and history and treatment (Creswell, 2013). To eliminate the first threat, the groups to which the research results could be generalized were limited, and care was taken to ensure that the students in the groups had similar characteristics in terms of socioeconomic background and academic performance. The study was conducted in a public school with two classrooms with similar physical features. In addition, applications in the experimental and control groups started and ended simultaneously. Classes in both groups were held in the morning. Although the fact that the study lasted six weeks and was not a complete experimental study is a factor that reduces external validity, the fact that fourth-grade students who have experienced all the achievements of primary school are included in the study minimizes this effect. In addition, the fact that the number of participants was close to each other is considered an important factor, and conducting complete experimental studies with large study groups at different grade levels in the future will make it possible to generalize.

More than one researcher measuring a phenomenon in the same way over the same period is defined as an indicator of internal reliability, whereas measuring the phenomenon over the same period is defined as an indicator of external reliability (Yıldırım & Şimşek, 2006). To ensure the internal and external reliability of the quantitative research, a model, participants, and data collection tools suitable for the research questions were determined. The researcher and an expert collaborated to score the collected data. In addition, the application process and data analysis steps are described in detail. Allocating two class hours for the pre-test and post-test and using the same test as both the pre-test and post-test can be listed as factors that increase internal reliability. However, the fact that two class hours are too much for a measurement process for students in this age group is an issue that needs to be considered. This is because there were three types of questions in the test and five questions from each type. Therefore, it is thought that preparing tests with fewer questions or focusing on a single type in future studies would be measures to increase internal reliability. In addition, the fact that the researcher provided training on the problem-posing approach to the teacher of the applied class and conducted a pilot study increases the external reliability.

In qualitative research, credibility instead of internal validity, transferability instead of external validity, consistency instead of internal reliability, and confirmability instead of external reliability are used (Yıldırım & Şimşek, 2006). Credibility refers to the trustworthiness of inferences drawn from data related to the studied phenomena or concepts. The researcher analyzed the data of two classroom teachers enrolled in postgraduate education to ensure credibility. Then a faculty member with expertize in mathematics teaching examined the analysis results. For example, the student who coded S4 said, "*It was enjoyable to solve the problem my friend posed*." This view was included in the "emotion-stimulating" category with coded words such as "fun, enjoyable, not bored"; however, it was discussed why this expression was not included in the "innovative" category. Since in the "innovative" category, the attachment to materials such as textbooks, notebooks, and projection devices was more prominent than students' emotions, it was decided to include this opinion, which appeals to students' emotions, in the "emotion-stimulating" category. This process (creating codes and categories together) can be considered an important objective criterion that increases credibility by reducing researcher bias. Transferability in qualitative research refers to the extent to which qualitative research findings can be generalized or transferred to other studies. To ensure transferability, the interview process was described in detail, and the students' views were included in the findings. Transparent disclosure of the duration and setting of the interviews and the necessary guidance contributed significantly to the transferability of the research results. Consistency deals with the extent to which the data reach the same conclusions as those in other research. To this end, the researcher and the two experts worked together to generate themes based on the codes inferred from the interviews. In cases of disagreement, reaching a common decision contributed to obtain consistent data. Finally, confirmability refers to the extent to which the study findings are free from the researcher's bias and align with other studies. To ensure confirmability, the preparation of the qualitative data collection tool, data collection, and data analysis were explained in detail, and a faculty member supervised the entire process. The fact that a faculty member and two classroom teachers continuing their postgraduate education participate in the study process is considered an important element that strengthens confirmability.

Ethical considerations

In this study, all rules stated to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions stated under the title "Actions Against Scientific Research and Publication Ethics", which is the second part of the directive, were not taken.

Ethical review board name: Afyon Kocatepe University Social and Humanity Sciences Scientific Research and Publication Ethics Committee

Date of ethics review decision: 15.10.2021

Ethics assessment document issue number: 2021/329

RESULTS

Findings in the First Stage of the Research

The results of the independent samples t-test conducted to determine whether there was a significant difference between the pre-test and post-test mean scores of the problem-posing skills of the experimental and control groups are shown in Table 6.

Table 6

Analysis of the Problem-posing Skills Pretest and Posttest Scores

Situation	Group	Test	n	*M*	sd	df	t	p
Structured	Experimental	Pretest	17	10.47	3.16	32	-0.615	0.543
	Control		17	11.05	2.35			
	Experimental	Posttest	17	13.88	0.99	32	4.611	0.000*
	Control		17	11.82	1.55			
Semi-Structured	Experimental	Pretest	17	10.64	3.69	32	-0.172	0.865
	Control		17	10.88	4.27			
	Experimental	Posttest	17	13.47	1.58	32	3.267	0.003*
	Control		17	11.23	2.33			
Free	Experimental	Pretest	17	10.23	3.21	32	0.091	0.928
	Control		17	10.11	4.22			
	Experimental	Posttest	17	13.05	1.51	32	2.954	0.006*
	Control		17	10.76	2.81			
Total points	Experimental	Pretest	17	31.35	8.20	32	-0.233	0.817
	Control		17	32.05	9.43			
	Experimental	Posttest	17	40.41	2.85	32	4.809	0.000*
	Control		17	33.82	4.87			

*p<0.05

In Table 6, no statistically significant difference was observed between the pretest scores of the experimental and control groups regarding the sub-dimensions and total score [t(32)=–0.615; t(32)=–0.172, t(32)=0.091; t(32)=–0.233, p>0.05]. According to these findings, the problem-posing skills of the students in the experimental and control groups were similar before the application.

In the analysis of posttest scores, a statistically significant difference was observed between the data of the experimental group and the data of the control group [t(32)=4.611; t(32)=3.267, t(32)=2.954; t(32)=4.809, p<0.05]. These findings show that teaching mathematics supported by problem-posing strategies effectively develops students' problem-posing skills.

Table 7 shows the dependent samples' t-test results to determine whether there is a significant difference between the problem-posing skills pre-test mean score and the post-test mean score of the experimental and control groups.

Table 7

Analysis of the Experimental and Control Group Pretest and Posttest Scores

Situation	Group	Test	n	*M*	sd	df	t	p
Structured	Experimental	Pretest	17	10.47	3.16	16	-5.010	0.000*
		Posttest	17	13.88	0.99			
	Control	Pretest	17	11.05	2.35	16	-1.176	0.257
		Posttest	17	11.82	1.55			
Semi-Structured	Experimental	Pretest	17	10.64	3.69	16	-3.699	0.002*
		Posttest	17	13.47	1.58			
	Control	Pretest	17	10.88	4.27	16	-0.466	0.647
		Posttest	17	11.23	2.33			
Free	Experimental	Pretest	17	10.23	3.21	16	-3.447	0.003*
		Posttest	17	13.05	1.51			
	Control	Pretest	17	10.11	4.22	16	-1.009	0.328
		Posttest	17	10.76	2.81			
Total points	Experimental	Pretest	17	31.35	8.20	16	-5.667	0.000*
		Posttest	17	40.41	2.85			
	Control	Pretest	17	32.05	9.43	16	-1.078	0.297
		Posttest	17	33.82	4.87			

*p<0.05

In Table 7, a statistically significant difference was observed between the pretest and posttest scores of the experimental group for the subdimensions and total score [t(16)=–5.010; t(16)=–3.699, t(16)=–3.447; t(16)=–5.667, p<0.05]. No significant difference was observed between the pretest and posttest scores of the control group [t(16)=–1.176; t(16)=–0.466, t(16)=–1.009; t(16)=–1.078, p>0.05].

These findings show that the activities carried out in the experimental group were effective in improving the problem-posing skills of the students. Teaching mathematics supported by problem-posing strategies significantly affected all three sub-dimensions and the total score.

Findings on the Second Stage of the Research

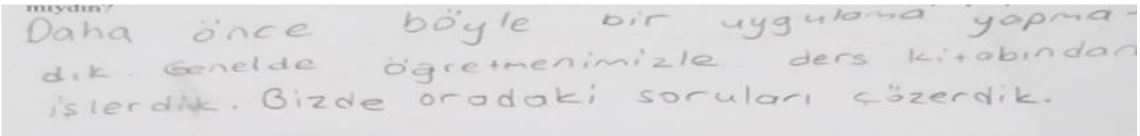
According to the first question in the interview form, students expressed opinions in three different categories (innovative, student-centered, and emotionally stimulating) about different aspects of teaching mathematics supported by problem-posing strategies. The findings are presented in Table 8.

Table 8

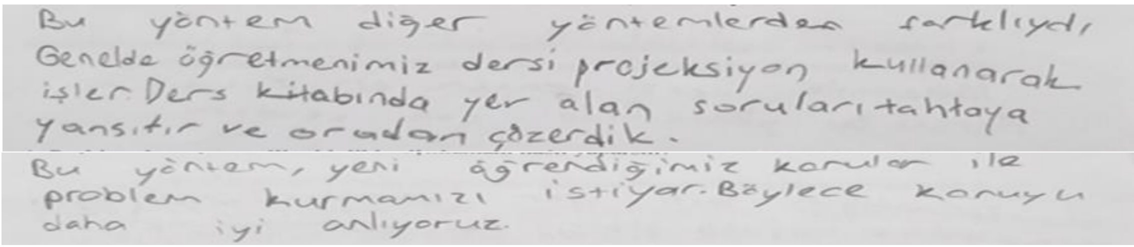
The Ways Teaching Mathematics Supported by Problem-posing Strategies Differs from Traditional Mathematics Teaching according to Students

Category	Code	Students
Innovative	Not traditional, no dependance on teaching materials	S2, S3, S4, S6, S7, and S8
Student-centered	Students taking an active role in learning, assuming the role of a teacher	S1, S4, S5, and S6
Emotionally stimulating	Fun, not boring	S1, S2, S3, S4, S5, S7, S8

According to Table 8, students whose views fell under the "innovative" category stated that they had not done such an application before. They noted that this method, unlike traditional applications, could be applied without relying on materials such as textbooks, notebooks, and projectors. (Figure 1).


mıydın?
Daha önce böyle bir uygulama yapmadık. Genelde öğretmenimizle ders kitabından işlerdik. Bizde oradaki soruları çözerdik.

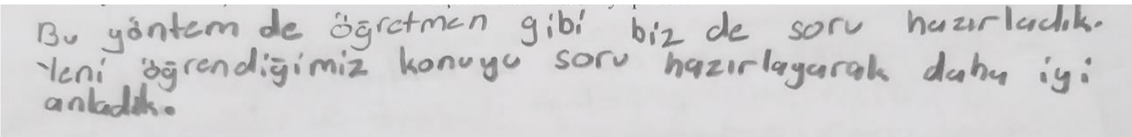
S2: "We have not done such activities before. Usually, our teacher uses the textbook to teach us a subject. We would do the tasks in the textbook. But this process (teaching Mathematics supported by problem-posing strategies) was very different."


Bu yöntem diğer yöntemlerden farklıydı, Genelde öğretmenimiz dersi projeksiyon kullanarak işler. Ders kitabında yer alan soruları tahtaya yansıtır ve oradan çözerdik.
Bu yöntem, yeni öğrendiğimiz konular ile problem kurmamızı istiyor. Böylece konuyu daha iyi anlıyoruz.

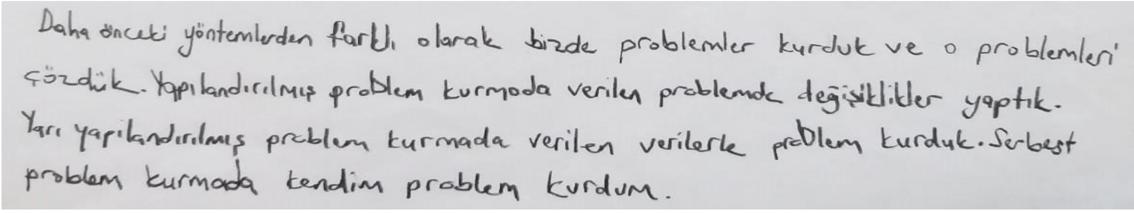
S3: "This method is different from other methods. Usually, our teacher uses a projector. We project the problems in the textbook onto the board and solve them in this way. In this method, we are expected to produce new problems using the new things we have learned. Thus, we can learn the subject better."

Figure 1. *Student Views of the Innovative Category*

Students whose views fell under the "student-centered" category stated that, unlike the previous methods, they freely posed and solved problems using this method. Since they felt like teachers and asked each other questions, they took an active role in the process and assumed the role of the teacher (Figure 2).


Bu yöntem de öğretmen gibi biz de soru hazırladık. Yeni öğrendiğimiz konuyu soru hazırlayarak daha iyi anladık.

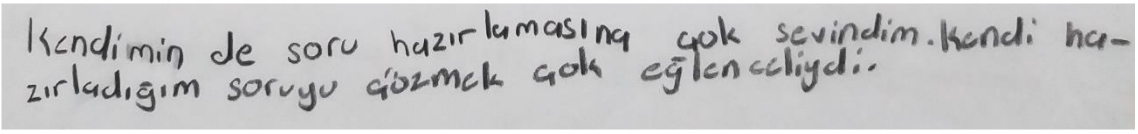
S1: "In this method, we produced problems like our teacher. We better learn new subjects by solving problems."


Daha önceki yöntemlerden farklı olarak bizde problemler kurduk ve o problemleri çözdük. Yapılandırılmış problem kurmada verilen problemde değişiklikler yaptık. Yarı yapılandırılmış problem kurmada verilen verilerle problem kurduk. Serbest problem kurmada kendim problem kurdum.

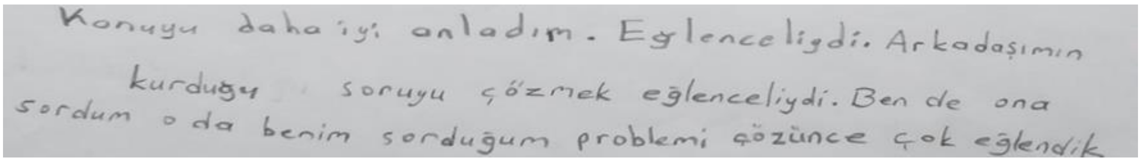
S5: "Unlike previous methods, we produced problems and solved them. In the structured problem-posing tasks, we made changes to the structured problems. In the semi-structured problem-posing tasks, we posed problems with given data. In free problem-posing tasks, we posed our own problems."

Figure 2. *Student Views of the Student-centered Category*

Students whose views fell under the "emotionally stimulating" category stated that they had fun because they prepared problems and asked each other about them. Unlike traditional practices (teacher writing questions on the board, students solving the problems on the board or in the textbook), they participated in the lesson without getting bored. They had fun because they were active (Figure 3).


Kendimin de soru hazırlamasına çok sevindim. Kendi hazırladığım soruyu çözmek çok eğlenceliydi.

S1: "I was very glad to produce a problem. It was very fun to solve the problem that I had produced."


Konuyu daha iyi anladım. Eğlenceliydi. Arkadaşımın kurduğu soruyu çözmek eğlenceliydi. Ben de ona sordum o da benim sorduğum problemi çözünce çok eğlendik.

S4: "I learned the subject better. It was fun. It was fun to solve the problems posed by my classmates. Then, I posed a problem for them to solve; we fun."

Figure 3. *Student Views of the Emotionally Stimulating Category*

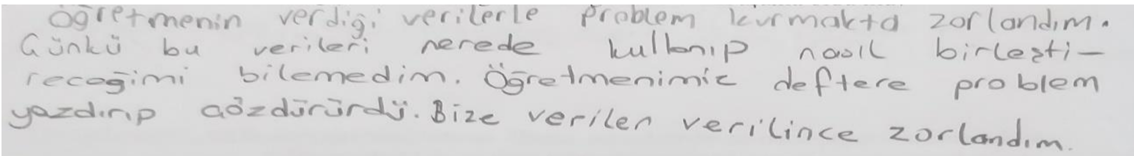
According to the second question in the interview form, students expressed their opinions in a single category regarding their difficulties while learning and teaching mathematics supported by problem-posing strategies. The findings are presented in Table 9.

Table 9

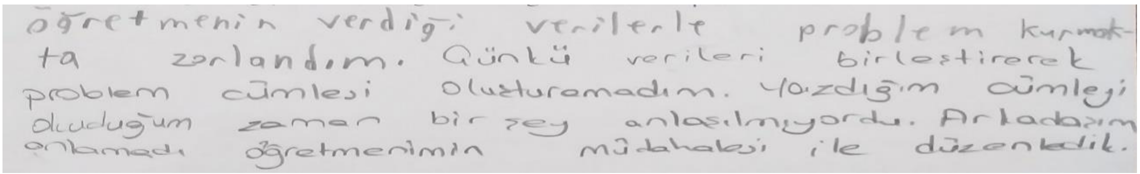
Stages/situations the students had difficulty with during teaching mathematics supported by problem-posing strategies

Category	Code	Students
Using strategies	Semi-Structured Problem-Posing Tasks	S7, S8
	Free Problem-Posing Tasks	S1, S4, and S6

According to Table 9, two students stated that they had difficulties combining the teacher's data and creating a meaningful problem sentence from these data because of the feature of the semi-structured problem-posing strategy. They said that their teachers always asked them to write problems in their notebooks. Therefore, they had difficulty combining data and information because they actively participated in this process. When they read the problems they wrote, they found them meaningless, irregular, or incomplete. With their teachers' intervention, they could transform the data and information into meaningful, solvable mathematical problems with rules (Figure 4).


Öğretmenin verdiği verilerle problem kurmakta zorlandım. Çünkü bu verileri nerede kullanıp nasıl birleştireceğimi bilemedim. Öğretmenimiz deftere problem yazdırıp çözdürürdü. Bize verilen verilince zorlandım.

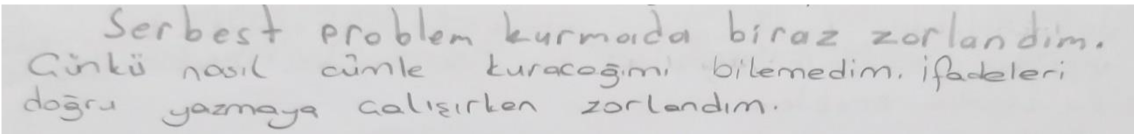
S7: "I had difficulty posing a problem using the data given by the teacher. because I did not know where to use the data. (In the past), our teacher would have us write problems in our notebooks and solve them. I had difficulty when we were given data to produce problems."


öğretmenin verdiği verilerle problem kurmakta zorlandım. Çünkü verileri birleştirerek problem cümlesi oluşturamadım. Yazdığım cümleyi okuduğum zaman bir şey anlaşılmıyordu. Arkadaşım anlamadı öğretmenimin müdahalesi ile düzenledik.

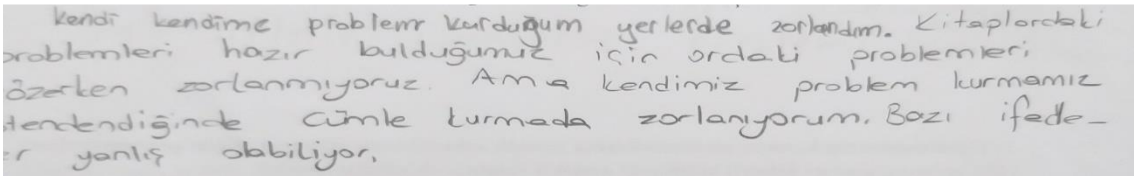
S8: "I found it difficult to produce problems with the data given by the teacher. Because I could not use the data properly to write a problem statement. When I read my problem statement, it was not understandable. My friend also did not understand it, so we edited it with my teacher's help."

Figure 4. *Students' Views on Semi-structured Problem-posing Strategies*

Three students stated that due to the nature of the free problem-posing strategy. In contrast, the students were left free to pose problems, and they had difficulty in combining the statements in the problem sentence in a meaningful way because they had no experience in how to form problem sentences other than the problem-solving and posing actions they were used to while posing problems. Because they were used to solve problems in textbooks, they had difficulty posing problems regarding a particular subject. Some students made ambiguities in their posed problems and could not use realistic data. They saw that the problems they had set up were unsolvable. They made the necessary corrections by obtaining help from their teachers so that the problems they formed were consistent with the mathematical principles and language. (Figure 5).


Serbest problem kurmada biraz zorlandım.
Çünkü nasıl cümle kuracağımı bilemedim. İfadeleri
doğru yazmaya çalışırken zorlandım.

S4: "I had a little difficulty with the free problem-posing tasks. Because I did not know how to write problem statements. I had a hard time writing the statements."


kendi kendime problem kurduğum yerlerde zorlandım. Kitaplardaki
roblemleri hazır bulduğumuz için ordaki problemleri
özerken zorlanmıyoruz. Ama kendimiz problem kurmamız
tendendiğinde cümle kurmada zorlanıyorum. Bazı ifade-
r yanlış olabiliyor.

S6: "I had difficulties with free problem-posing situations. We do not have any difficulties (with the tasks) in the textbook because it instructs us about what to do. However, I had a hard time when we were asked to pose problems ourselves. Some statements were wrong."

Figure 5. *Student Views on the Free Problem-posing Strategy*

DISCUSSION and RESULTS

This study was conducted to reveal the effect of mathematics teaching supported by problem-posing strategies on the problem-posing skills of fourth-grade students and to obtain students' opinions about this method.

The first stage of the research concluded that teaching supported by problem-posing strategies was more effective in developing students' problem-solving skills than the control group. In addition, while the teaching supported by problem-posing strategies in the experimental group improved students' problem-posing skills, the teaching based on the current curriculum in the control group did not improve students' problem-posing skills. The emergence of this success in the experimental group can be attributed to the planned and qualified integration of problem-posing into mathematics lessons. The problem-forming learning model designed by Örnek and Soylu (2021) improves conceptual learning, ensuring the solvability of problems. It encourages the correct use of mathematical language and grammar rules and demonstrates the need for such models. Baumanns and Rott (2022) conducted 36 task-based interviews with pre-service elementary and secondary mathematics teachers who were given two structured problem-posing tasks to describe and analyze structured problem-posing processes. At the end of the study, they defined five types of activities (situation analysis, variation, generation, problem-solving, and evaluation). It was determined that this definition provided a better understanding of problem-posing processes in general. Turhan and Güven (2014) found that the problem-posing approach conducted with sixth-grade middle school students improved students' problem-posing skills, supporting our study's results. In addition, related studies have revealed that problem-posing-based teaching has a positive effect on the problem-solving success of students with different levels of number perception (Işık et al., 2012) and academic success in teaching integers (Özdemir & Şahal, 2018). However, Güzel and Biber (2019) found that the problem-posing approach to teaching inequalities did not significantly affect academic achievement. Despite this result, Cantürk Günhan et al. (2019) revealed in their meta-analysis that problem-posing-based mathematics teaching positively and significantly affects student achievement. In addition, it has been concluded that problem-posing approaches supported by digital environments contribute to the development of students' problem-solving, problem-posing, and creative thinking skills (Kanbur Tekerek & Argün, 2019; Nuha et al., 2018; Suarsana et al., 2019; Sung et al., 2016). Akben (2018; 2019) determined that the problem-posing approach used in science teaching increased pre-service teachers' problem-solving skills and academic achievement. In addition, he found that pre-service teachers became aware of their level of knowledge about this approach and stated that it would contribute to their professional development. The results of these studies show that the problem-posing approach is effective in improving students' problem-solving, problem-posing, academic achievement, and creative thinking skills in both mathematics and other courses, which shows how important it is to be aware of and use the problem-posing approach in every grade and level. However, the few studies in which problem-posing is considered a method reveal that more research should be conducted to guide teachers in effectively performing this method. In addition, it should not be forgotten that studies should be conducted to include problem-posing more in curriculum content (Divrik et al., 2020; Jia & Yao, 2021; Zhang & Cai, 2021). It would also be helpful to conduct studies that focus on the different skills of students using this method at the primary school level, include teacher training, and provide guidance to classroom teachers on how to incorporate this method into their lesson plans.

When an evaluation was made regarding the three sub-dimensions of problem-posing, both in the inter-group comparison and the intra-group evaluation, the teaching supported by problem-posing strategies carried out in the experimental group was effective in the development of the experimental group's problem-posing skills in all three sub-dimensions. The lack of significant improvement in the control group can be attributed to the teacher's continued teaching based on the textbook. The fact that fifth-grade students' success in problem-posing was positively affected by the structured and semi-structured problem-posing activities prepared for the acquisition of "solving and constructing problems requiring operations with natural numbers" supports the results of our study. (Şakar, 2018). However, there are studies in the literature that reveal that teachers, pre-service teachers, and students have difficulty with different problem-posing strategies (Köken & Gökkurt-Özdemir, 2018; Kubanç & Ayaz, 2019; Ngah et al., 2016; Özgen et al., 2017; Silber & Cai 2016; Ulusoy & Kepceoğlu, 2018). For example, Silber and Cai (2016) found that pre-service teachers were more successful in revealing mathematical concepts in structured problem-posing activities than in free. However, there are also studies in the literature that reveal that teachers and pre-service teachers have the most difficulty in structured problem-posing activities (Köken & Gökkurt-Özdemir, 2018; Kubanç & Ayaz, 2019). Another study determined that pre-service teachers had more difficulty with semi-structured problem-posing activities than free problem-posing activities. Pre-service teachers were more successful in free problem-posing because they could construct free problems specific to their desired context without any restrictions. However, they had difficulty posing problems in semi-structured problem-posing activities because they were partially restricted (Mersin & Akkaş, 2023). These studies show that different problem-posing strategies have unique characteristics and should be systematically practiced. However, the insufficient number of objectives (Özgen et al., 2017), limited time, textbook content limitations, and teachers' inability to perform problem-posing activities (Lee et al., 2018) present some difficulties. This approach can be integrated into teachers' professional development courses to overcome the difficulties of using different strategies and make more precise distinctions (Bicer et al., 2020; Passarella, 2021). The results of this study concluded that problem-posing strategies can be integrated into elementary school mathematics courses.

In the second stage of the current research, because of analyzing the students' responses to the first question, it was determined that teaching mathematics supported by problem-posing strategies is an "innovative, student-centered, and emotionally stimulating" application. As is known, we encounter math at every stage of our lives, either directly or indirectly. Understanding and using math is becoming increasingly important every day because we need math to solve many problems encountered in everyday life. Recent changes in curricula emphasize the need to develop learning environments that help students learn math more easily (MoNE, 2017; 2018). In addition, relevant studies have underlined the importance of promoting innovative, student-centered, and fun learning environments that improve students' math skills (Divrik, 2019; Güneş et al., 2011; Karasu Avcı & Ketenoğlu Kayabaşı, 2019; Keklik, 2018). In this sense, in contrast to classical learning methods, the study participants thought that teaching mathematics supported by problem-posing strategies was innovative. Kontorovich (2020) found that problem-posing triggers a sense of innovation in producing good problems, which supports the results of this study. Similarly, the study by Turhan and Güven (2014) found that the problem-posing method was more effective than traditional textbook-based teaching. Furthermore, the participants of this study actively participated in performing problem-posing tasks, which resulted in their assessment of the method as a student-centered method. Likewise, Erdem and Soylu (2019) found that learning environments incorporating various teaching-learning materials (computer-aided applications, educational games, concrete teaching materials, cartoons, and discussion in collaborative groups relating to everyday life) boosted student engagement, which is consistent with our findings. Kilpatrick (1987) found that the mathematical tasks performed by students consisted of problems produced by their teachers or included in textbooks. However, when the students posed their problems, solved the problems produced by their classmates, and realized that they, too, could pose problems, they started to have more fun. In addition, the students who produced their problems enjoyed solving the problems produced by themselves and their classmates. However, when the studies on the emotional effects of problem-posing recently are examined, it is emphasized that more research should be conducted on this issue (Cai & Leikin, 2020; Guo et al., 2020; Parhizgar et al., 2021; Schindler & Bakker, 2020). Therefore, emotional reactions are also important in cognitive skills, and more studies have investigated emotional reactions in problem-posing processes (Cai & Leikin, 2020).

The analysis of the students' responses to the second interview question revealed that students had the most difficulty in "semi-structured and free problem-posing" strategies while teaching mathematics supported by problem-posing strategies. Similarly, Kırnap-Dönmez (2014) revealed that prospective primary school mathematics teachers were more successful in structured problem-posing tasks than in semi-structured and free problem-posing tasks, which is consistent with our findings. Because structured problem-posing activities are more accessible, understandable, and doable than other problem-posing activities, they can easily construct problems by changing the information, data, and statements on the problem or adding new information. Studies in the relevant literature supported this result (Divrik, 2019; Kılıç, 2014; Kırnap-Dönmez, 2014; Tertemiz & Sulak, 2013).

In the semi-structured problem-posing strategy, students had difficulty combining data and information into meaningful mathematical problems in accordance with the rules. The conclusion of Ulusoy and Kepceoğlu (2018) that middle school mathematics teacher candidates made similar mistakes (mathematical language or ambiguities) supports the results of this study. Divrik et al. (2020) found that semi-structured problem-posing activities were the most common in primary school mathematics textbooks. According to this finding, students engage in semi-structured problem-posing activities more than other problem-posing strategies. However, this finding is not a result that can be effective in developing students' semi-structured problem-posing skills on its own. There are other variables that teachers (organization, designing, evaluation, quality problem-posing, negative impact on exams) and students (low-quality problem-posing, lack of experience, difficulty using language, lack of confidence) have to overcome in problem-posing tasks (Li et al., 2020; Xie & Masingila, 2017). Therefore, when conducting semi-structured problem-posing activities, the teacher should first pose sample problems, encourage the students, and provide a road map on what to do, which may help the students to pose more qualified problems.

It was revealed that students had difficulty in free problem-posing activities because they did not have previous experience in topic selection, determining data, determining skills, determining the number of operations, determining the limitations of the problem, and bringing together all data in accordance with mathematical principles. Although eighth-grade students feel comfortable in free problem-posing activities, and this reflects positively on their problem-posing success (Karahan Doğuz & Genç, 2023), studies have revealed that teachers give more space to structured and semi-structured problem-posing activities during the lesson (Işık & Kar, 2012) and that free problem-posing situations are a more challenging task compared to other strategies (Ngah et al., 2016; Özgen et al., 2017). Lack of experience, lack of content knowledge, lack of curriculum knowledge, not recognizing students' cognitive levels, and difficulties in writing problem texts are cited as the reasons for difficulties in free problem-posing tasks (Şengül & Katrancı, 2015). In this context, the fact that students had difficulty in free problem-posing activities is consistent with the results of these studies. In future studies, quantitative studies can be conducted to reveal which strategies students have more difficulty with at the primary school level, and longitudinal studies to overcome these difficulties will make important contributions to the literature.

LIMITATIONS AND RECOMMENDATIONS

This study was limited to 34 4th-grade students and 8 interviewees studying in a public school in Türkiye. The sample size was too small to draw a general conclusion that teaching mathematics supported by problem-posing strategies can improve students' problem-posing skills. because it was conducted with a quasi-experimental design. In addition, the application of the study was limited to 6 weeks and 30 lessons. This period is also not enough to make a general assessment that problem-posing-based teaching always increases students' problem-posing skills. Because in these six weeks, only lessons on fractions were conducted. Research over a more extended period, including other subjects, may eliminate this limitation in this respect. One final limitation of the current research is related to the subject taught in the process. The researcher used the studied method only in the subjects of "Fractions" and "Fraction Operations." In addition, the teacher's prejudice toward going beyond the teaching practices they are used to at first can be considered an uncontrollable limitation.

Considering these limitations, we can make some suggestions. We found that teaching mathematics supported by problem-posing strategies had a positive effect on students' problem-posing skills. However, students still had difficulty in the semi-structured and free problem-posing tasks. Therefore, we recommend that teachers use all three problem-posing strategies effectively. For this purpose, it may be helpful to first provide in-service training to teachers about problem-posing pedagogy and to conduct practical studies. Teachers can follow a path from simple to more complex applications while conducting classroom problem-solving activities. First, new problems can be constructed on a problem already solved, such as changing data or adding further information. Then, problem sentences that are left incomplete can be completed, or problems can be constructed using data such as tables, figures, and graphs. When a certain level of competence is reached in these applications, problems involving selected topics and operations can be constructed. Teachers who apply this process in their classes, for example, can enable their students to solve problems in accordance with this stage. In addition, future studies should focus on the effects of teaching mathematics supported by problem-posing strategies on the mathematical skills of students at different grade levels. Since it was evaluated that integrating the problem-posing strategy into subjects (measurement, data collection, numbers) that require four operations would be more effective, studies covering these learning areas, as well as studies to be conducted in the field of geometry learning, can shed light on the related literature. It would be more appropriate to design these studies using accurate experimental designs. Creating these experimental studies as longitudinal studies may be more qualified to observe the development over time. In addition, by studying with larger study groups and achieving a longer implementation period may generalize this positive effect. It is also important to conduct studies that reveal the emotional impact of problem-posing activities.

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