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Research Article

Examination of Types of Mathematical Connections that Pre-service Mathematics Teachers Use in Instructions

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Abstract:

This study aims to examine the types of mathematical connections that pre-service mathematics teachers use in their instructions and their opinions on the mathematical connection. For this purpose, a total of 60 pre-service mathematics teachers made lesson plans related to a learning objective from the Turkish middle school mathematics curriculum. The case study design, which is one of the qualitative research methods, was adopted in the study. Video recordings, field notes and an opinion form were used as data collection tools. The data were analyzed using content analysis method. Results of the study show that almost all of the pre-service teachers have an understanding of mathematical connections. It was determined that the types of connections between concepts, between different representations and connections with real life were prominent, while the type of association with different disciplines was less common during the instructions. Connections were mainly made by giving examples through verbal expressions. These results provide an important perspective on the mathematical connection skills of pre-service primary mathematics teachers. Suggestions for improving pre-service teachers' mathematical association skills were presented acording to the results of the study.

Keywords: Mathematical connection, pre-service mathematics teachers, case study

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INTRODUCTION

In mathematics, operations, concepts, systems, and structures are interrelated. Mathematics is seen as a sequential and cumulative discipline, as it has a structure in which concepts builds on each other in that previous knowledge and concepts form the basis of subsequent concepts and systems. (Bingölbali & Coşkun, 2016). These interactions between different mathematical concepts and systems emphasize mathematical connections. The mathematical connections help learners to see mathematics as a holistic disipline, rather than as a collection of independent and discrete parts (Mwakapenada, 2008). For example, when we consider that every square is also a rectangle and every rectangle is also a parallelogram, there is a relationship between the concepts of square, rectangle, and parallelogram in terms of their common properties. If this relationship is recognized, then these geometric objects are evaluated with a holistic approach rather than separately.

Hiebert and Carpenter (1992) defined the mathematical connection as a part of a network structured like a spider web, where "the junctures, or nodes can be thought of as pieces of represented information, and threads between them as the connections or relationships" (p. 67). Mathematical connections, which are characterized as bridges between mathematical ideas, can also be defined as components of a schema or connected schema groups in a mental network (Eli & Mohr-Schroeder, 2013). Mathematical connection skill, which helps to establish these links, play an important role in mathematics education (National Council of Teachers of Mathematics [NCTM] 2005). Mathematical connection also plays an important role in associating mathematics with different disciplines and daily life (Özgen, 2016). Mathematical connections include both the connections between mathematical concepts and systems and the connections between mathematics in different disciplines and real life. García-García and Dolores-Flores (2018) state that the connections between different subjects of mathematics are considered intra-mathematical connections; if these connections are related to different fields or daily life, they are considered extra-mathematical connections. There are different classifications in the literature for mathematical connection, which has been found to contribute positively to the learning and instructions (NCTM, 2005). In the theoretical framework section, we explain which mathematical connection classification was adopted within the scope of the study.

Theoretical Framework

There is a strong relationship between mathematical connections and mathematical understanding (Cai & Ding, 2017; García-García & Dolores-Flores, 2018; Hiebert & Carpenter, 1992). Mathematical connections make mathematical understanding powerful and deep in the process of constructing knowledge (Skemp, 1989). Mathematical connections can be made not only between mathematical concepts, ideas, or operations, but also with everyday life, prior knowledge, other disciplines, the past, and the future (Begg, 2001; Presmeg, 2006). Mathematical connections strengthen mathematical understanding,

ensure consistent answers, utilize different representations, work inside and outside mathematics, and are influenced by students' belief systems. (Businskas, 2008; Evitts, 2004; García-García & Dolores-Flores, 2018; Özgen, 2013). In this context, Bingölbali and Coşkun (2016) proposed a theoretical framework for systematic classification of mathematical connections to be used more effectively in mathematics education. According to the theoretical framework, mathematical connections are discussed under four categories: the connections between mathematical concepts, the connections between different representations of the concept, the connections with real life, and the connections with different disciplines.

Connections Between Mathematical Concepts

Skemp (1986), pointing out the difficulty of making a precise definition, tried to explain the term "concept" with the process of abstraction that results in a product. “Mathematical concepts consist of mathematical (mental) objects and the relationships among those objects.” (Simon, 2017, p. 121). Therefore, in mathematics teaching, it is necessary to establish connections between mathematical concepts to create or improve schemas, which are defined as conceptual structures according to the cognitive approach If the connection occurs between different mathematical concepts or expressions, it should be considered as a connection between concepts (Bingölbali & Coşkun, 2016). Considering that each rectangle is a special form of a parallelogram, the relationship between the concepts of parallelogram and rectangle can be given as an example of connections between concepts.

Connections Between Different Representations of a Concept:

Making connections between different representations of a concept is defined as re-representing the same concept in different forms such as words, graphs, tables, and equations (Prain & Waldrip, 2006). Representing mathematical objects in multiple ways plays an important role in mathematical understanding (Dreher & Kuntze, 2015; Van de Walle, Karp, & Bay-Williams, 2012). Flexibility in transitions between different representations facilitates mathematical learning (Ainsworth, Bibby, & Wood, 1998). In this context, Bingölbali and Coşkun (2016) considered different representations of concepts and the connections between them within the scope of mathematical connections and gave the value table of the dependent and independent variables in the expression whose algebraic representation is $f(x) = x^2$ and the graph on the analytical plane as examples of different representations.

Connections with Real Life

The importance of establishing meaningful relationships between real life and mathematics in mathematics teaching is collectively suggested in the literature (Julie, 2002; Ministry of National Education [MoNE], 2018; NCTM, 2000; Simon & Cox, 2019). This means making use of contexts and examples that students encounter or are likely to encounter in

their daily lives. Bingölbali and Coşkun (2016) evaluated verbal examples given from daily life as well as addressing a concept in context under the type of connections with real life.

Connections with Different Disciplines

Interdisciplinary work is of great importance for the development of 21st-century skills such as questioning, critical thinking, problem-solving, creative and innovative thinking (English, 2016). Nowadays, innovations in medicine, engineering, industry, and many social fields occur through projects that bring together experts from different disciplines. This situation makes it necessary to adopt an interdisciplinary approach both in mathematics teaching and in the teaching of other courses to prepare students for the future. Bingölbali and Coşkun (2016) discussed the situation of using a concept or relationship in a different discipline in teaching a mathematical concept in the category of associating it with different disciplines. According to the theoretical framework, these connections can be made by using the context of a different discipline or by giving a verbal example in another discipline.

Importance, Purpose and Research Questions of the Study

Mathematical connection has been studied with teachers (Mhlolo, Venkat, & Schafer, 2012; Rodríguez-Nieto, Rodríguez-Vásquez, & Font, 2020), pre-service teachers (Eli, Mohr-Schroeder, & Lee, 2013; Özgen, 2019; Rodríguez-Nieto, Rodríguez-Vásquez, & García-García, 2021a; Zengin, 2019) and students at different grade levels (García-García & Dolores-Flores, 2018; 2021). The studies conducted with teachers, pre-service teachers, and students based on making connections between different representations (Mhlolo, Venkat, & Schafer, 2012; Moon, Brenner, Jacob, & Okamoto, 2013), real-life connections (Özgeldi & Osmanoğlu, 2017; Pirasa, 2016), connections between concepts or different disciplines in the context of specific topics (Dolores-Flores, Rivera-Lopez, & Garcia-Garcia, 2019; García-García & Dolores-Flores, 2021; Rodríguez-Nieto, Rodríguez-Vásquez, & García-García, 2021a), and examining participants' views on making associations (Özgeldi & Osmanoğlu, 2017; Özgen, 2013). In these studies, participants' mathematical skills in different contexts were examined. In addition, there are also studies investigating the mathematical connections used by teachers in the subjects they teach in the classroom environment (Jaijan & Lopia, 2012; Rodríguez-Nieto, Rodríguez-Vásquez, & Font, 2020). It is important to determine the mathematical connections made in the lessons and how they are used in teaching (Dolores-Flores, Rivera-Lopez, & Garcia-Garcia, 2019). This situation draws attention to future teachers' undergraduate level. Because the decisions that teachers make in their professional lives cannot be considered independent from the education and training processes during their undergraduate education (Ünlü & Sarpkaya Aktaş, 2017). However, the studies investigating the mathematical connection used in the instructions of pre-service mathematics teachers are limited. Özgen (2019) conducted a study with 19 pre-service mathematics, 11 pre-service physics, and 10 pre-service chemistry teachers and focused on

the participants' ability to connect mathematics with different disciplines in the activities they designed. Özgeldi and Osmanoğlu (2017) examined how pre-service mathematics teachers made connections between real life and mathematics through short videos and reports of the participants. No study in this direction was found in the relevant literature. In this context, the main aim of the study is to determine which types of mathematical connections primary school mathematics pre-service teachers use in the instruction they designed in line with the achievements in the secondary school mathematics curriculum. In addition, it is to examine which methods they used in this process and their views on mathematical connections. For these purposes, the following research questions were tried to be answered:

1. Which types of mathematical connections did primary pre-service mathematics teachers use in the instruction that they designed in line with the achievements of the mathematics curriculum?
2. Which methods did the primary pre-service mathematics teachers use in the mathematical connections they made during the instruction they designed in line with the achievements of the mathematics curriculum?
3. What are the views and opinions of pre-service mathematics teachers on mathematical connections?

METHOD

Research Model

The case study model, which is one of the qualitative research designs, was used in the study. According to Yin (1994), researchers examine a case in-depth in its natural environment and have the opportunity to obtain detailed and holistic knowledge about the case in case studies.

Participants

The participants of the study were determined by purposive sampling method. In purposive sampling, the researcher selects cases that are rich in information regarding the determined research questions and purpose, which will provide the opportunity for detailed investigation (Patton, 2002). The participants of the study consisted of 60 (44 were female (73.33%) and 16 male (26.66%) junior pre-service mathematics teachers who were from a faculty of education in İstanbul. The pre-service teachers were coded as PST1, PST2, ..., PST60. The participants had taken courses related to content knowledge, pedagogical content knowledge, and pedagogical knowledge in the undergraduate curriculum such as general mathematics, abstract mathematics, linear algebra, calculus 1-2-3, analytic geometry, instructional technologies and material design, algebra teaching, mathematics teaching in secondary school, mathematics teaching methods 1, teaching principles and

methods, educational psychology. Participants signed an informed consent form before the implementation.

Data Collection Tools and Implementation

The data were collected within the scope of the Mathematics Teaching Methods 2 course. While the Mathematics Teaching Methods 1 course curriculum in the program is theoretical, the Mathematics Teaching Methods 2 course generally focuses on practical activities. In this context, pre-service teachers were asked to plan a 12-20 minutes teaching activity for an objective from the secondary school mathematics curriculum. Then they acted and thaught as if they were in a real classroom environment. During the instruction, it was stated that the other participants in the class would assume that they are a primary school student and act as it. In this way, we aimed to capture an atmosphere similar to that of a real primary classroom environment in the micro-instruction. The first and second authors took part in the instruction as observers. The implementation lasted for 14 weeks. In the first week, information about the research was presented to the participants, an informed consent form was given to the participants who wanted to be included in the application. In each following week, lecture sections of 4 or 5 participants were included. In order not to disrupt the natural flow, participants were informed that the data would be used within the scope of a research; however, it is not stated that the scope of research is related to mathematical connection.

The data were collected through video recordings, researchers' observation notes, and an open-ended questionnaire. The instruction carried out by the pre-service teachers was videotaped, and short transcriptions of the content were made by labeling the important times in these recordings.

Data Analysis

A total of 920 minutes (15 hours and 20 minutes) of video recordings were analyzed. In addition, comparisons were made with the observation notes that were taken during the teaching sessions. The content analysis method was used to analyze the data through video recordings and open-ended questionnaires. Content analysis is a repetitive and systematic data analysis technique in which data are reduced to certain categories within a certain rule (Weber, 1990). The content analysis method is a technique that can be used not only on texts but also in the analysis of videos (Büyüköztürk et al., 2017). The theoretical frameworks of Bingölbali and Coşkun (2016) and García-García and Dolores-Flores (2018) form the basis for the data analysis. Figure 1 summarizes the implementation and data analysis process.

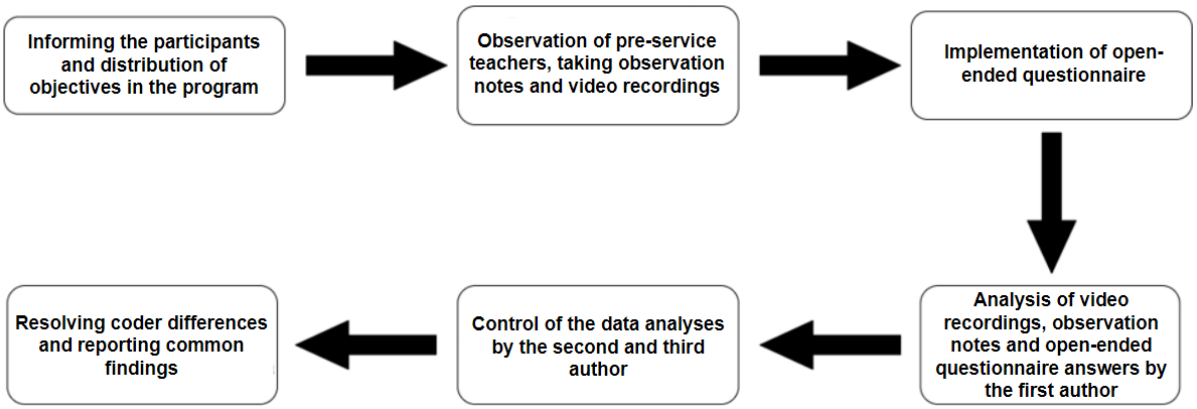

Figure 1. The implementation and data analysis process

Validity and Reliability

To ensure validity and reliability in qualitative research, strategies such as detailed narration, long study process, triangulation, direct quotation, and determination of intercoder agreement should be used (Creswell & Miller, 2000; McMillan & Schumacher, 2014). In this respect, the data obtained from the study were detailed, a long implementation process of 14 weeks was carried out, different data collection tools were used, the participants' statements were conveyed with direct quotations, and intercoder reliability was examined during the data analysis process. According to the intercoder reliability formula of Miles and Hubermann (1994), the agreement between the coders was found to be 87.65%.

Ethical considerations

Since the data were collected in the spring semester of 2019, no ethics committee report was obtained. All data acquired, including the video recordings and participants' consent documents, were securely housed on the first researcher's personal computer, fortified by stringent password protection measures.

RESULTS

Findings Related to the Types of Connections Used by Pre-Service Mathematics Teachers

In this section, the findings related to the types of mathematical connections used by pre-service mathematics teachers in their teaching sections are presented. Most of the participants used one or more types of connections in their instructions. Table 1 summarizes the types of mathematical connections used by pre-service teachers.

Table 1

Mathematical Connections Used by Pre-Service Mathematics Teachers

Intra/extra-mathematical	The type of connection	f	%
Intra-mathematical connections	Connections between concepts	18	30
	Connections between different representations of concept	32	53,33
Extra-mathematical connections	Connections with real life	26	43,33
	Connections with different disciplines	4	6,66
No connection was made		7	11,66

According to the findings, 80% of the pre-service teachers used mathematical connections, while 11.66% of them did not use any type of mathematical connection in their instruction sessions. Among the pre-service teachers who used mathematical connections in their lectures, 30% made connections between concepts, 53.33% made connections between different representations, 43.33% made connections with real life, and 6.66% made connections with different disciplines. In addition, we observed the pre-service teachers who utilized two or three different types of connections in their instructions. While 28.33% of the pre-service teachers utilized two different types of connections, 20% of them utilized three different types of connections in their teaching sessions.

PST11 included connections between concepts, connections between different representations, and connections with real life in the teaching section in which she presented for the achievement of "calculating the area of a circular region and a circular region slice" of Grade 7 in the mathematics curriculum. PST11 referred to the connection between different representations by using circles and circle slices obtained from colored cartons. She also used the connection with real life by projecting visuals such as bagel, money, car tire, cake surface, lifebuoy, and moon on the screen. Nevertheless, PST11 wanted to draw attention to the difference between the concepts of circle and circular region with the projected objects. Lastly, she made the connection between concepts by reaching the area of the circular region by making use of the relationship between the circular region, the slices of the circular region, and the parallelogram consisting of these slices as seen in Figure 2.

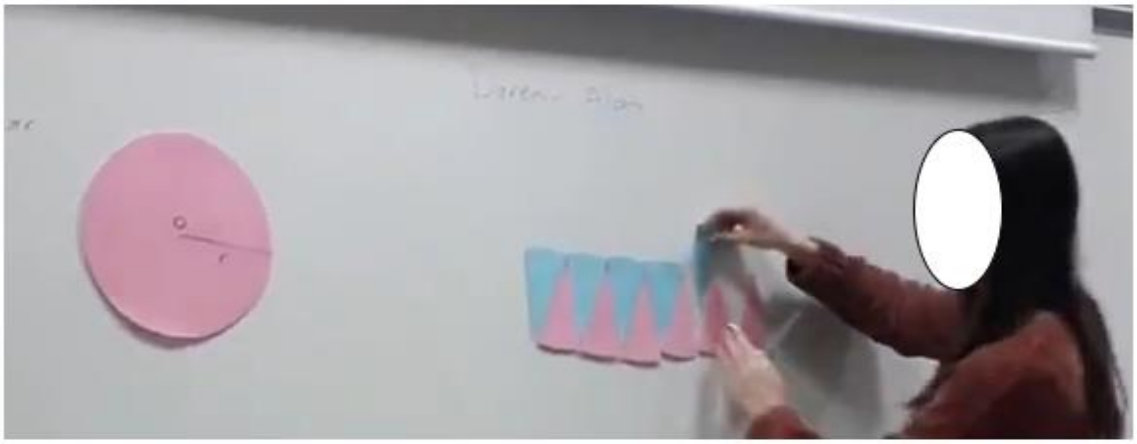

Figure 2. A section from the instruction of PST11

In the instruction designed for the objective "determining the basic elements of a right circular cylinder, constructs it and draws its expansion" of 8th grade, PST5 used two different types of connections. PST5 made an introduction to the lesson as follows:

> *"What you see in my hand is a right prism (showing models of prisms). We could define a prism as follows: We call the objects whose base is a polygonal region and whose side faces are rectangular regions right prisms. How did we name them? Whichever polygonal region was their base, that is what we named our prism. What you see in my hand is a square right prism. This is a hexagonal right prism. This is a triangular prism. Now I will ask you a question. We have chosen these bases as polygonal regions. If we had chosen a circular region, what kind of an object would we get?"*

PST5's effort to establish a connection between the circular cylinder and prisms showed that she used the connection between concepts. Besides, her presentation on the expansion of the cylinder with the help of the physical material that she designed from expansion tiles, a rope, and cardboard and dynamic geometry software showed that she used the connection between different representations. Figure 3 shows the carton materials and the reflection dynamic geometry software.

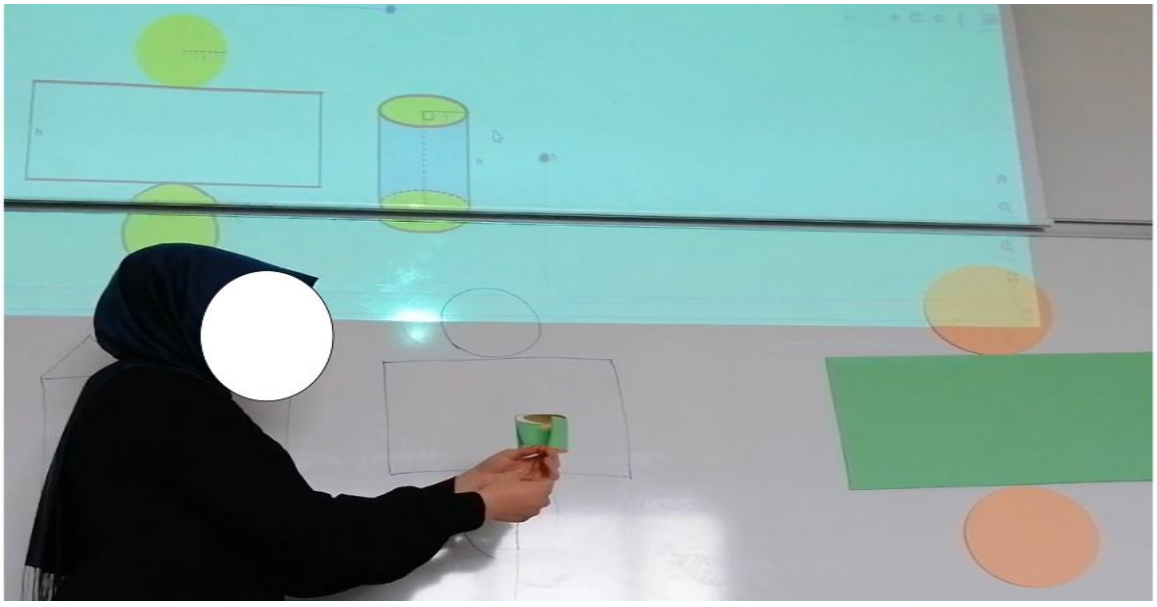

Figure 3. A section from the instruction of PST5

Findings Related to the Methods Used by Pre-Service Mathematics Teachers in Mathematical Connections

In this section, we presented findings related to making connection methods of pre-service mathematics teachers according to the types of mathematical connections.

The connection methods of the pre-service teachers who made connections between concepts were evaluated in the categories of verbal expressions, drawing or representation, and utilizing physical materials. Table 2 shows the preservice teachers' methods of making connections between concepts.

Table 2

The Methods of Connections Between Concepts Used by Pre-Service Mathematics Teachers

The methods of connections between concepts	**f**
Through verbal expressions	10
Through drawing or demonstrations	6
Through physical materials	2

Table 2 shows that pre-service teachers made connections between concepts mainly with verbal expressions. For example, on the subject of ratio and proportion, PST57 made connections between concepts with verbal expressions by saying that:

> *"Since our topic was ratio and proportion, do you remember rational numbers? We had a topic called "expansion and simplification" in rational numbers. Therefore, we had a topic called equivalent fractions. What happens when we multiply the numerator and denominator by the same number in these fractions? We were doing the expansion, right?"*

In addition, we observed that the preservice teachers made connections by presenting physical materials (PST5), using drawings (PST24, PST43), or projecting visuals related to the concepts (PST32). Figure 4 shows the diagonal drawings made by PST24 to connect the sum of interior angles in polygons with triangles, the drawing of the cube expansion made by PST43 to connect the surface expansion of a rectangular prism with a cube, and the visuals reflected by PST32 about the area connection of a triangle. The pre-service teachers who made connections between concepts through drawing or demonstration and with the help of physical materials also benefited from connections between different representations. However, the representations that pre-service teachers performed were not related to the target concept, but to another concept related to that concept.

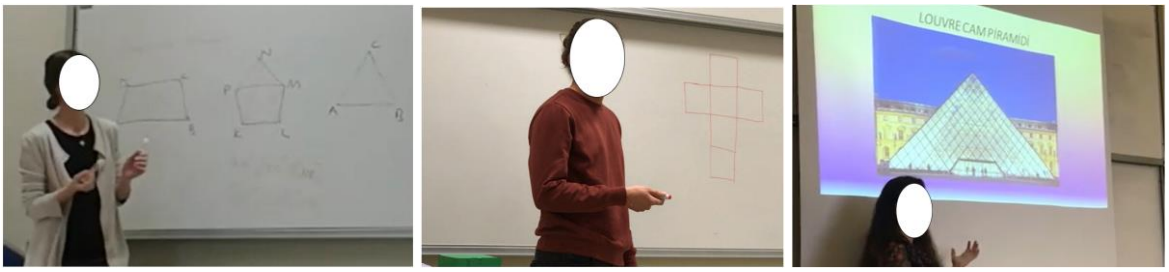

Figure 4. Sections from the instruction of PST24 (left), PST43 (middle), and PST32 (right)

Table 3 summarizes methods for the connections between the different representations that pre-service teachers made for the focus objective in the instruction.

Table 3

The Methods of Connections Between Different Representations of Concept Used by Pre-Service Mathematics Teachers

The methods of connections between different representations of concept	**f**
Through physical materials	28
Through dynamic software or interactive environments	13
Through drawing or demonstrations	9

We observed that the pre-service teachers who made connections between different representations mainly used physical materials. We also observed that the pre-service teachers used carton mockups, algebra tiles, paper clips, compasses, scales, balances, balls, and rods for the focused objective in the teaching sessions. For example, PST33, who carried out an activity using carton cups, chickpeas, and beans related to the GCF (greatest common factor)-LCM (least common multiple) topics in the 8th grade, stated that:

> *"Now I have 9 beans and 6 chickpeas. Now I want to distribute them equally. Logically, if I distribute them to both of them one by one, am I distributing them equally? ... Then can you put 9 cups for 9 beans? 6 cups for 6 chickpeas. One bean and one chickpea in each cup. Now I want you to do the following. I want to distribute them equally, but this time I've added a second condition. I want the number of cups to be the minimum..."*

After the physical materials, we observed that pre-service teachers made connections between different representations with the help of dynamic software or interactive environments. Pre-service teachers mainly made connections related to the target objective with the help of GeoGebra. Figure 5 shows the instruction sections of PST3 showing the angles of prisms and PST49 showing the center of gravity of a triangle.

Figure 5. Sections from the instruction of PST3 (left) and PST49 (right)

In the teaching sessions, connections between different representations were also made through drawings or demonstrations. We observed that the pre-service teachers who used this method benefited from making connections by projecting ready-made visuals, watching video recordings, or making drawings by hand. As can be seen in Figure 6, while PST20, who designed instruction on expressing the rule of number patterns of the 7th grade with letters, projected a shape pattern on the board, PST19, who designed instruction on constructing the bisector, bisector and height in a triangle, made drawings on the board with the help of a ruler, protractor and compass.

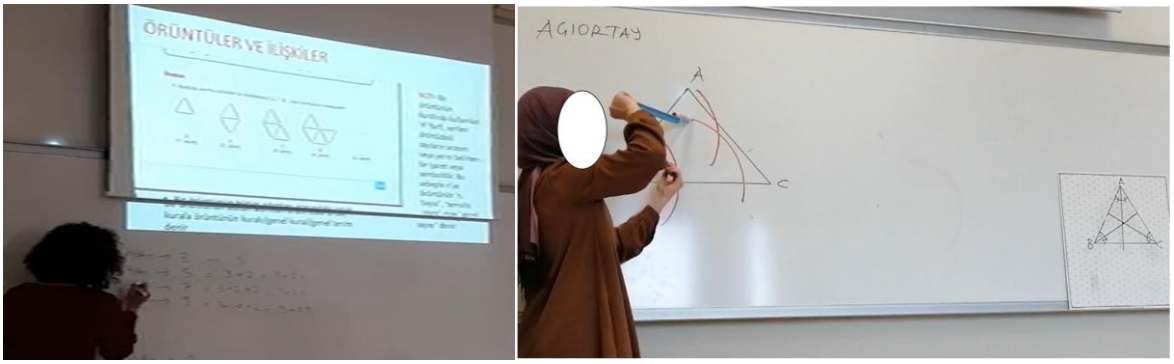

Figure 6. Sections from the instruction of PST20 (left) and PST19 (right)

The pre-service teachers' methods of making connections with real life were categorized as through verbal expressions, history of mathematics, demonstrations, and physical materials. Table 4 presents the methods of making connections with real life.

Table 4

The Methods of Connections with Real Life Used by Pre-Service Mathematics Teachers

The methods of connections with real life	**f**
Through verbal expressions	13

Through demonstrations	8
Through physical materials	6
Through history of mathematics	1

The pre-service teachers who made connections with real life mainly used verbal expressions, demonstrations, and physical materials. We observed that the pre-service teachers who used verbal expressions or representations presented examples of physical objects, experiences, or experiences from daily life related to the mathematical concepts or situations. In the 6th grade, PST12, who designed instruction related to the attainment related to addition and subtraction of fractions, used the following statements verbally:

> *"Now, what about addition with fractions in daily life... Can you think of any event? Let's think about it. Maybe when buying fabric, right? For example, I want half of this fabric batch. When I go back later, I say, I want one-fourth of it. I have a certain amount of fabric. I can find out how much the batch of whole I've bought by adding it up."*

PST12 connected a shopping context in daily life with addition in fractions by using verbal expressions.

The pre-service teachers also made connections with real life by using demonstrations. They presented visuals or video demonstrations of the related mathematical concepts or objects to the students. As the demonstrations were mostly presented to the students with the help of technology, it was observed that the available relevant visuals were presented physically to the class. Regarding the 6th grade attainment "Drawing a circle and recognizing its center, radius, and diameter", PST16 started the lesson by projecting an image of a Ferris wheel. In 8th grade, in order to make a connection with the topic of reflection in transformation geometry, PST15 presented a photograph of a landscape reflected in water to the class. Figure 7 shows the presentations of the pre-service teachers.

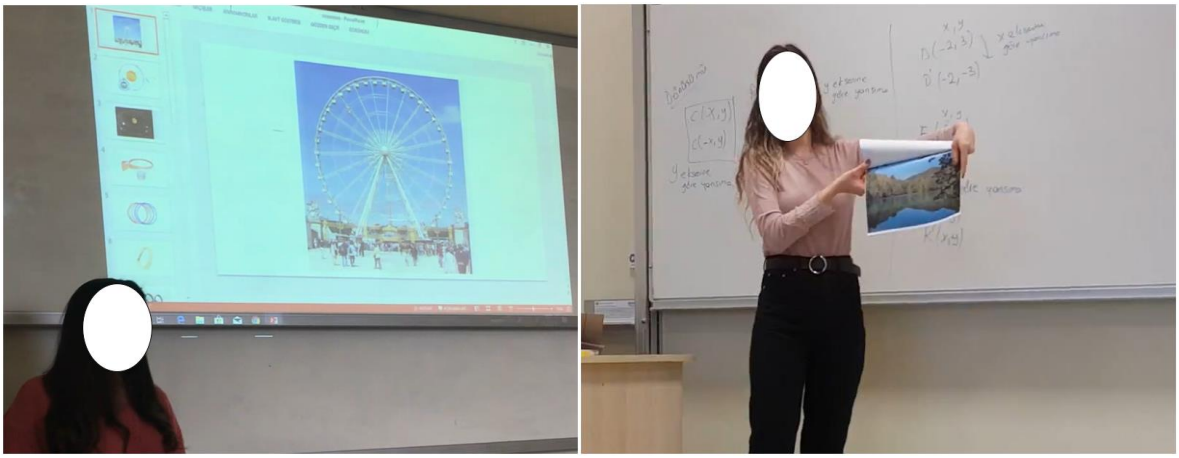

Figure 7. Sections from the instruction of PST16 (left) and PST15 (right)

In the connections with real life, we found that in addition to verbal expressions and demonstrations, the pre-service teachers also made connections through physical materials. We observed that the pre-service teachers who used this method concretized a daily life situation related to the targeted outcome with the help of the tools that they brought to the classroom. In 7th grade, PST26 used a material representing a scale and marbles to display the example of a scale related to the attainment of "Understanding the principle of conservation of equality". Similarly, PST59, who wanted to make connections between positive and negative integers and temperature and coldness related to the attainment of "Comparing and ordering integers" in 6th grade, brought two bottles to the classroom, one with hot (+4) and the other with cold (-5) water. Figure 8 shows the moments of the connections made by the pre-service teachers through physical materials.

Figure 8. Sections from the instruction of PST26 (left) and PST59 (right)

The only pre-service teacher who made a connection with real life by using the history of mathematics was PST10. She used the following expressions by connecting the 8th-grade attainment of "Forming the Pythagorean relation and solving related problems" with land calculations in the ancient Egyptian period:

> *"Pythagoras named this relation after himself. Pythagoras was a Greek mathematician. He lived in 500 BC. This relation was formed as follows: There was a lot of flooding in the Nile River in Egypt. Therefore, the boundaries of the fields and lands were always changing. For this reason, they used the length of the hypotenuse of a right triangle with a known right side length to determine the boundaries. And hypotenuse was the name of Pythagoras' wife."*

The least common type of connection used by pre-service teachers in the teaching sessions was connection with different disciplines. The teaching sessions in which we observed this connection were categorized according to the field of connection. The connections made by pre-service teachers were evaluated in the categories of science, finance, geography, and cryptology. Table 5 presents these disciplines.

Table 5

The Methods of Connections with Connections with Different Disciplines Used by Pre-Service Mathematics Teachers

The methods of connections with different disciplines	**f**

Science	2
Finance	1
Geography	1
Cryptology	1

The pre-service teachers made connections with different disciplines by using verbal expressions. Examples of mathematical concepts or situations were given from the fields of physics, chemistry, biology in science, interest calculations in finance, and continents in geography, and an activity was carried out by using encryption. In 8th grade, PST7, who connected exponential expressions with both science and finance, used the following statements:

> *"We use exponential numbers to identify the number of bacteria or the rate at which they multiply. We use them to calculate the diameters of planets or measure the distance between two planets. We use them in interest calculations. We use them to express large numbers or, in chemistry, to show the diameter of atoms."*

Similarly, PST21 verbally connected the addition and subtraction of fractions in 5th grade with the topic of continents in geography:

> *"The areas covered by the continents of the world, Africa and Asia, are expressed in fractions. How can we find the total area? The fraction of one-fifth represents Africa, that is, the area it covers. The fraction of three-tenth represents the Asia continent in terms of its area. How can I find the total area of these two continents? By addition. Now in this lesson, we will see how to add these two different fractions."*

Making connections with real life by using physical materials, PST59 also designed an encryption activity that combines the alphabet with the topic of comparing integers. The rules of his activity are as follows:

- Write your own name on a paper,
- Number it from the top in the order of the letters in the name starting from 1,
- Number it from the bottom in the order of the letters in the name starting from -1,
- Compare the numbers above and below the letters,
- Whichever number is larger, move through the alphabet as much as that number and change the corresponding letter.

In her activity, PST59 reversed the idea of comparing the encryption process and the decryption process with integers and applied it in the classroom. Figure 9 shows the password "NVC" obtained for the word "MUZ" and the word "FATMA" obtained as a result of the solution of the password "GCVPE".

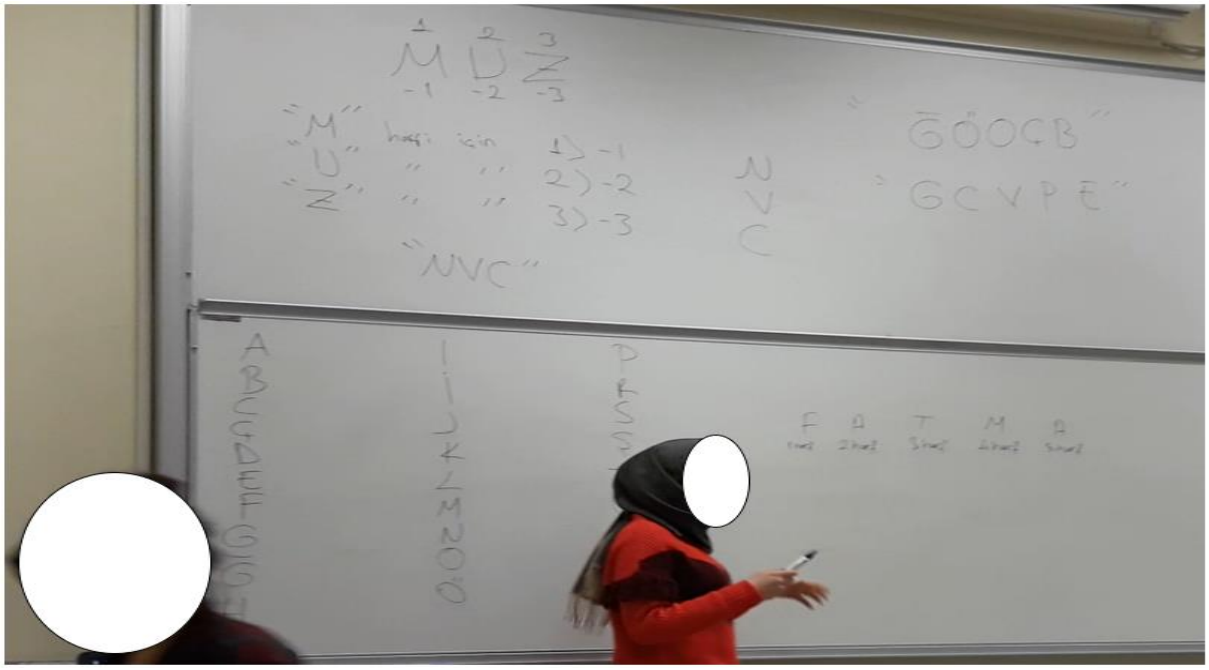

Figure 9. Sections from the instruction of PST59

Findings Related to Views of Pre-service Mathematics Teachers on Mathematical Connections

The findings of the study revealed that almost all of the pre-service teachers considered mathematical connection as one of the basic components of mathematics education. Table 6 summarizes pre-service teachers' views on mathematical connection.

Table 6

Views of Pre-service Mathematics Teachers on Mathematical Connections

Mathematical connection	**f**
Making connections with daily life	48
Making connections between mathematical concepts or topics	30
Making connections with different disciplines	11
Mathematical symbolism/demonstratiton	2
No idea	1

According to the findings, the majority of the pre-service teachers perceived mathematical connection in the context of making connections with daily life. Besides, the majority of pre-service teachers with this view reflected the idea of making abstract mathematical concepts or topics concrete through objects and experiences in daily life. They

emphasized that students would learn more easily, meaningfully, and permanently in this way. For example, PST22 reported that connecting with daily life would facilitate learning:

> *"We as teachers should concretize mathematics more and transfer it to students. While concretizing, we have to connect mathematics with something. I think that if it is understood that the problems that students face in their daily lives are actually mathematical, it will be easier for the students."*

Similarly, PST30 also reflected the idea of concretization and mentioned the contribution of mathematical connection to meaningful learning:

> *"Mathematical connection is making connections between mathematics and daily life. It is the answer to the question of what these will do for us. When this connection is established, abstract concepts become concrete."*

PST 17 also mentioned the permanence of learning and stated that:

> *"When I think of mathematical connection, the intertwining of daily life and mathematics comes to my mind. When we explain a topic in mathematics and present it with a similar example from daily life, I think its retention increases even more."*

After the connections with daily life, pre-service teachers emphasized the connections between mathematical concepts and subjects for mathematical connection the most. The pre-service teachers who drew attention to this issue referred to the holistic structure of mathematics and the fact that mathematical subjects are the basis of each other. They reported that the connection between concepts would contribute to the interest in the course, the instruction, and the understanding of the subject matter. PST12, who also addressed mathematical connection in the context of making connections with daily life, pointed to the holistic structure of mathematics by stating that:

> *"Mathematics is a cumulative science. The reason for this is the mathematical relationships between the subjects. A teacher should establish these relationships very well so that the student knows that the subjects are not independent of each other and that he/she should know the related subjects."*

PST42 also emphasized that different subjects in mathematics are the basis of each other and reported that:

> *"What is fundamental in mathematical connection is that a subject is also encountered when we move on to other subjects. Almost all mathematics topics are interrelated and it becomes impossible to understand the other topic without knowing one of them."*

Similarly, PST35 referred to the holistic structure of mathematics and mentioned that mathematical connection contributes to the teaching and understanding of the course stating that:

> *"Mathematics is a whole. The subjects in mathematics complement each other. Each of them is related to each other. For example, the topics of percentages, ratio-proportion, and fractions are related to each other. This facilitates both teaching and student understanding."*

Explaining mathematical connection as establishing links between daily life, mathematical subjects, and different disciplines, PST43 reflected his thoughts on interest in the lesson and ease of understanding. In light of these findings, the pre-service teachers who

handled mathematical connections in the context of the connections between concepts are thought to be making consistent explanations. The statements of PST43 are as follows:

> *"Mathematical connections can be made with daily life or between different subjects. Connections can be made with previous subjects of mathematics or even with the content of a different course. This increases the interest in the course and makes it easier to understand."*

Seven of the eleven pre-service teachers who evaluated mathematical connection in the context of making connections with different disciplines also included both making connections with daily life and making connections between mathematical concepts or subjects in mathematical connection. The other four pre-service teachers considered making connections with daily life as well as making connections with different disciplines as part of the mathematical connection. PST13 explained mathematical connection as the relationship established with both daily life and different disciplines and exemplified mathematical connection in the context of culture and art:

> *"Mathematical connection is the establishment of a relationship between concepts related to mathematics and daily life or other disciplines to make them more concrete. Mathematical connections can also be made with things that are part of humanity, such as culture and civilization. For example, calculating how many kilos of meat each of 7 people gets from a calf slaughtered on the Eid Al Adha is a connection between culture and mathematics. On the other hand, a person drawing a picture calculating perspective is a connection between art and mathematics."*

S31 also mentioned all three types of connections stating that:

> *"Relating mathematical concepts within themselves, with other disciplines and with daily life is called mathematical connection."*

It is noteworthy that all of the pre-service teachers who evaluated mathematical connections as making connections with different disciplines also pointed to other types of connections.

Finally, we observed that two pre-service teachers addressed mathematical connections in the context of mathematical symbolism or demonstration. Among these pre-service teachers, PST11 mentioned making connections between mathematical concepts or subjects and different representations in addition to making connections with daily life:

> *"Mathematical connection is the connection of mathematical concepts with each other. Formulas are also the connection of symbols. For example, showing natural numbers and integers in a set. Making use of ratio and proportion when explaining percentages."*

The other pre-service teacher, PST6, evaluated mathematical connection only in the context of symbolism:

> *"Mathematical connection is the expression of concepts in mathematics with symbols and numbers."*

One pre-service teacher did not express an opinion about mathematical connection.

As a result, pre-service teachers have various approaches and evaluations toward mathematical connection. They emphasized the importance of connecting mathematical concepts with daily life and stated that this would facilitate the learning process. At the same

time, pre-service teachers think that mathematical connections could contribute to the understanding of the connections between concepts and support the holistic structure of mathematics. In addition, they also suggested that making connections with different disciplines could enrich students' mathematical understanding.

DISCUSSION

This study aimed to examine which types of mathematical connections pre-service mathematics teachers use in the instruction that they design in line with the attainments of the middle school mathematics curriculum, which methods they use while making mathematical connections, and their views on mathematical connections. The video recordings of teaching sessions of the pre-service teachers were analyzed both simultaneously and after the recording. In this section, the findings of the study are discussed together with the results in the literature.

It was revealed that almost all of the pre-service teachers participating in the study had an understanding of mathematical connections. According to the results, the connection between concepts, between different representations, and with real life was prominent in the teaching sessions of the pre-service teachers, while the type of connections with different disciplines was less common. The pre-service teachers mainly made connections by giving examples through verbal expressions in their instructions. The opinions of the pre-service teachers showed that they had different approaches and evaluations of mathematical connections.

Among the types of mathematical connections used by pre-service teachers in the instructional sessions, connections between different representations came to the fore. This result is similar to the results of Agry et al. (2023), Rodríguez-Nieto, Rodríguez-Vásquez, and García-García (2021a; 2021b). Agry et al. (2023) concluded that pre-service primary school teachers made connections between different representations. Rodríguez-Nieto, Rodríguez-Vásquez, and García-García (2021a; 2021b) found that the type of connection between different representations was the type of connection that pre-service teachers used the most in the derivative subject. It is also noteworthy that pre-service teachers mainly utilized physical materials and technology when making connections between different representations. In addition, the opinions of the pre-service teachers revealed that they had an understanding of the mathematical connection that aimed to concretize the abstract concepts of mathematics. These findings show that pre-service teachers have an awareness of the power of physical materials and technology, and have the skills to use these resources. On the other hand, the findings of the study contradict the studies of Moon, Brenner, Jacob, and Okamoto (2013) and Quilang and Lazaro (2022), who reported that pre-service teachers had difficulty in making connections between different representations. This may be related to the mathematical concept or topic to which the connection is related. While Moon, Brenner, Jacob, and Okamoto (2013) examined different representations of conic curves,

Quilang and Lazaro (2022) examined the types of connections of pre-service teachers about statistics and probability.

Another type of connection that the pre-service teachers used the most in their teaching sessions was the connection with real life. The participants tried to establish the connection of mathematical concepts with daily life mostly through verbal expressions. However, real-life connections were also made using technology and physical materials. In addition to these findings, the theme of making connections with daily life came to the fore in pre-service teachers' views on mathematical connections. In light of this result, it can be said that pre-service teachers reflect their opinions that this type of connection is an effective teaching strategy. The findings of the study on making connections with real-life support the results of Özgeldi and Osmanoğlu (2017) and Özgen (2013). Özgeldi and Osmanoğlu (2017) found that pre-service teachers were able to make explicit connections with real life and that most of them thought that all subjects of mathematics could be connected with real life. Özgen (2013) observed that pre-service teachers mainly addressed mathematical connection in the context of connection with real life. Although pre-service teachers dealt with mathematical connections in the context of making connections with real life, their connection in the teaching sessions remained superficial. We observed that pre-service teachers who made connections with real life mostly gave examples from objects or situations in daily life and did not make in-depth connections. These findings support the results of Pirasa (2016) and Aguirre et al. (2012), who concluded that pre-service teachers made superficial and simple connections between mathematics and real life. The fact that the connections made by pre-service teachers with real life are superficial may be related to the examples of connections in school mathematics and textbooks in their education life. This is because mathematics textbooks tend to give a mathematical definition and then emphasize the connections superficially by giving examples of where they are encountered in real life (Yeniterzi & Işıksal-Bostan, 2015).

After connections between different representations and connections with real life, the pre-service teachers used connections between concepts the most. Pre-service teachers made connections between concepts mainly through verbal expressions. We also observed that mathematical concepts were connected through drawing or demonstration in the teaching sessions. In addition, pre-service teachers' views on mathematical connections revealed that the connections between mathematical concepts or topics are related to the holistic structure of mathematics. They pointed out that all subjects are interdependent, addressing the various mathematical topics based on the curriculum. Agry et al. (2023) emphasized that mathematical concepts become inseparable topics with a holistic understanding through mathematical connection. It can be stated that the findings of the study regarding the connection between concepts support this idea. The results of the study are in line with the results of similar studies in the literature (Agry et al., 2023; Dışbudak Kuru & Işıksal Bostan, 2023: Rodríguez-Nieto, Rodríguez-Vásquez, & Javier García-García, 2021b). Dışbudak Kuru and Işıksal Bostan (2023) found that pre-service teachers tried to make as many connections

as possible between mathematical concepts to appeal to students' thinking in their teaching rehearsals. Similarly, Rodríguez-Nieto, Rodríguez-Vásquez, and Javier García-García (2021b) concluded that inter-conceptual connections were prominent in pre-service teachers' mathematical connections about derivatives.

The least common type of connection in the teaching sessions was connections with different disciplines. We observed that pre-service teachers made connections between concepts, between different representations, and with real life, but they preferred the interdisciplinary connection type less than the others. It can be said that this result supports the results of Dışbudak Kuru and Işıksal Bostan (2023) and Özgen (2013; 2019). Dışbudak Kuru and Işıksal Bostan (2023) concluded that pre-service teachers made connections with daily life, but the connections with different disciplines that they made were limited. Özgen (2013) also revealed that the contexts used by pre-service teachers in making connections with different disciplines were limited. In another study by Özgen (2019), while the use of mathematics in other disciplines was dominant, the use of other disciplines in mathematics education remained narrow. One of the possible reasons for this result might be the fact that making connections is seen as identical to concretization and visualization, as also revealed in the opinions of pre-service teachers. However, it is noteworthy that the pre-service teachers who addressed mathematical connections in the context of connections with different disciplines also emphasized other types of connections. In other words, pre-service teachers who have an idea about making connections with different disciplines are also aware of other types of connections. It can be said that this result indicates that making connections with different disciplines also provides opportunities for other types of connections. In parallel to this opinion, Dışbudak Kuru and Işıksal Bostan (2023) concluded that pre-service teachers who made progress in making connections with different disciplines also made progress in making connections with daily life.

CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS

This study, which examines in detail the types of mathematical connections used by pre-service mathematics teachers in their teaching sessions, revealed that connections made between different representations were prominent. While pre-service mathematics teachers frequently preferred connections with real life and connections between concepts, they used connections with different disciplines less frequently. In addition to the examples given superficially with verbal expressions, it is also noteworthy that pre-service teachers used physical materials and technology effectively while making mathematical connections. This study provides an important perspective on the types of mathematical connections used by pre-service mathematics teachers.

In light of the results of the study, we suggest including more practices and rehearsals on the mathematical connection in the undergraduate preperation of pre-service mathematics teachers. In addition, mathematical connections should be used in the courses that pre-service teachers take to strengthen their theoretical and practical background in

mathematical connections. This study is limited to the instructional sessions designed by pre-service teachers for a single outcome they selected in the middle school mathematics curriculum. In this respect, the mathematical connection strategies used by pre-service teachers in their teaching sessions designed for different learning domains and objectives may differ. Nevertheless, we recommend conducting research examining which types of materials are used in mathematical connections and the roles of these materials. Another limitation of the study is related to in-class interactions. In the videotaped teaching sessions, the focus was on the instruction of pre-service teachers rather than student-student interactions. In this respect, research can be conducted to examine in-class interactions and which types of connections can be used in different learning domains and objectives.

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Data Availability Declaration

Data Availability Upon Formal Request:

While the primary datasets utilized in this study are not publicly accessible due to certain constraints, they are available to researchers upon a formal request. The authors have emphasized maintaining the integrity of the data and its analytical rigor. To access the datasets or seek further clarifications, kindly reach out to the corresponding author. Our aim is to foster collaborative academic efforts while upholding the highest standards of research integrity.

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All authors, [Muhammet ŞAHAL], [Mustafa DOĞAN], and [Ahmet Şükrü ÖZDEMİR] contributed equally to this work. They collaboratively handled the conceptualization, methodology design, data acquisition, and analysis. Each author played a significant role in drafting and revising the manuscript, ensuring its intellectual depth and coherence. All authors have thoroughly reviewed, provided critical feedback, and approved the final version of the manuscript. They jointly take responsibility for the accuracy and integrity of the research.

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